ECE 302 - Linear System Analysis

Homework #3

Due Date: September 20, 2023

1. Lathi & Green problem 1.5-3
2. Lathi & Green problem 1.7-7

Remark: Book problem statements are attached.

3. MatLab Introduction:
   (a) Get access to MatLab (or some equivalent software package). See the course outline for more information on this.
   (b) Read the MatLab overview in Section B.7 of your text.
   (c) Plot the function
       \[ x(t) = \begin{cases} 
       |t| & : t \in [-1, 1] \\
       e^{-|t|} & : \text{otherwise}.
       \end{cases} \]

   using MatLab.
   (d) Compute the energy in \( x(t) \) using MatLab. Check your answer by hand.

Remark: Always present your simulations results in a clear, concise and professional manner. This means:

1. Provide the source code.
2. Title all plots and label all axes on all plots.
3. If more than one plot is on a single graph, supply a legend.

An unintelligible presentation will not be graded and result in zero credit.

4. Complex Arithmetic: Let \( z_1 = 4e^{-j\frac{\pi}{4}} \) and \( z_2 = 1 - j \). Compute the following:
   (a) \( z_1 + z_2 \)
   (b) \( z_1 z_2^* \)
   (c) \( z_2 / z_1^* \)

Put your final answer in rectangular form.
1.5-3 (a) Determine even and odd components of the signal \( x(t) = e^{-2t}u(t) \).
(b) Show that the energy of \( x(t) \) is the sum of energies of its odd and even components found in part (a).
(c) Generalize the result in part (b) for any finite energy signal.

1.5-4 (a) If \( x_e(t) \) and \( x_o(t) \) are even and the odd components of a real signal \( x(t) \), then show that\[ \int_{-\infty}^{\infty} x_e(t)x_o(t) \, dt = 0 \]
(b) Show that \[ \int_{-\infty}^{\infty} x(t) \, dt = \int_{-\infty}^{\infty} x_e(t) \, dt \]

1.5-5 An aperiodic signal is defined as \( x(t) = \sin(\pi t)u(t) \), where \( u(t) \) is the continuous-time step function. Is the odd portion of this signal, \( x_o(t) \), periodic? Justify your answer.

1.5-6 An aperiodic signal is defined as \( x(t) = \cos(\pi t)u(t) \), where \( u(t) \) is the continuous-time step function. Is the even portion of this signal, \( x_e(t) \), periodic? Justify your answer.

1.5-7 Consider the signal \( x(t) \) shown in Fig. P1.5-7.

(a) Determine and carefully sketch \( v(t) = 3x(-(1/2)(t+1)) \).
(b) Determine the energy and power of \( v(t) \).
(c) Determine and carefully sketch the even portion of \( v(t) \), \( v_e(t) \).
(d) Let \( a = 2 \) and \( b = 3 \); sketch \( v(at+b) \), \( v(at) + b \), \( av(t+b) \), and \( av(t)+b \).
(e) Let \( a = -3 \) and \( b = -2 \); sketch \( v(at+b) \), \( v(at) + b \), \( av(t+b) \), and \( av(t)+b \).

1.5-8 Consider the signal \( y(t) = \frac{1}{5}x(-2t-3) \) shown in Fig. P1.5-8.

(a) Does \( y(t) \) have an odd portion, \( y_o(t) \)? If so, determine and carefully sketch \( y_o(t) \). Otherwise, explain why no odd portion exists.
(b) Determine and carefully sketch the original signal \( x(t) \).

1.5-9 Consider the signal \(-\frac{1}{2}x(-3t+2) \) shown in Fig. P1.5-9.

(a) Determine and carefully sketch the original signal \( x(t) \).
(b) Determine and carefully sketch the even portion of the original signal \( x(t) \).
(c) Determine and carefully sketch the odd portion of the original signal \( x(t) \).

1.5-10 The conjugate symmetric (or Hermitian) portion of a signal is defined as \( w_{cs}(t) = (w(t) + w^*(-t))/2 \). Show that the real portion of \( w_{cs}(t) \) is even and that the imaginary portion of \( w_{cs}(t) \) is odd.

1.5-11 The conjugate antisymmetric (or skew-Hermitian) portion of a signal is defined as \( w_{ca}(t) = (w(t) - w^*(-t))/2 \). Show that the real portion of \( w_{ca}(t) \) is odd and that the imaginary portion of \( w_{ca}(t) \) is even.

1.5-12 Define \( w(t) = e^{j(t+\pi/4)} \).
(a) Referring to the definition in Prob. 1.5-10, determine \( w_{cs}(t) \). Express your simplified answer in standard rectangular form.
(b) Referring to the definition in Prob. 1.5-11, determine \( w_{ca}(t) \). Express your simplified answer in standard polar form.
1.7-3 Two inputs, temperature $T(t)$ and wind speed $V(t)$, produce an output, wind chill $W(t)$, according to $W(t) = 35.74 + 0.6215T(t) - 35.75[V(t)]^{0.16} + 0.4275T(t)[V(t)]^{0.16}$. The independent variable here is time, $t$. Answer the following questions yes or no, and provide mathematical justification for each answer.

(a) Is this system BIBO-stable?
(b) Is the system memoryless?
(c) Is the system causal?
(d) For simplicity, let the wind speed be constant, $V(t) = kV$. Thus, $W(t) = k_1 + k_2T(t)$ for some constants $k_1$ and $k_2$. Is this simplified system linear?
(e) For simplicity, let the temperature be constant, $T(t) = kT$. Thus, $W(t) = k_3 + k_4[V(t)]^{0.16}$ for some constants $k_3$ and $k_4$. Is this simplified system linear?

1.7-4 Input voltage $x(t)$ applied to an inverting op-amp follower circuit produces output $y(t)$ according to

$$y(t + t_p) = \begin{cases} 
-V_{\text{ref}} & x(t) > V_{\text{ref}} \\
V_{\text{ref}} & x(t) < -V_{\text{ref}} \\
x(t) & \text{otherwise}
\end{cases}$$

where op-amp reference voltage $V_{\text{ref}}$ and propagation delay $t_p$ are both positive constants. Answer the following questions yes or no, and provide mathematical justification for each answer.

(a) Is this system BIBO-stable?
(b) Is the system causal?
(c) Is the system invertible?
(d) Is the system linear?
(e) Is the system memoryless?
(f) Is the system time invariant?

1.7-5 Repeat Prob. 1.7-4 for a system with input $x(t)$ that produces output $y(t)$ according to

$$y(t + 1) = \begin{cases} 
-2x(t) & \text{when } x(t) \geq 0 \\
0 & \text{otherwise}
\end{cases}$$

1.7-6 Repeat Prob. 1.7-4 for a system with input $x(t)$ that produces output $y(t)$ according to

$$y(t - 1) = \begin{cases} 
x(t - 1) & \text{when } x(t) \geq 0 \\
x(t - 2) & \text{otherwise}
\end{cases}$$

1.7-7 Repeat Prob. 1.7-4 for a system that multiplies a given input by a ramp function, $r(t) = tu(t)$. That is, $y(t) = x(t)r(t)$.

1.7-8 Repeat Prob. 1.7-4 for a system with input $x(t)$ that produces output $y(t)$ according to

$$y(t) = \frac{d}{dt}x(t - 1)$$

1.7-9 Repeat Prob. 1.7-4 for a system with input $x(t)$ that produces output $y(t)$ according to

$$y(t) = \begin{cases} 
x(t) & \text{if } x(t) > 0 \\
0 & \text{if } x(t) \leq 0
\end{cases}$$

1.7-10 A continuous-time system is given by

$$y(t) = 0.5 \int_{-\infty}^{\infty} x(\tau)[\delta(t+\tau) - \delta(t)]d\tau$$

Recall that $\delta(t)$ designates the Dirac delta function.

(a) Explain what this system does.
(b) Is the system BIBO-stable? Justify your answer.
(c) Is the system linear? Justify your answer.
(d) Is the system memoryless? Justify your answer.
(e) Is the system causal? Justify your answer.
(f) Is the system time invariant? Justify your answer.

1.7-11 For a certain LTI system with the input $x(t)$, the output $y(t)$ and the two initial conditions $q_1(0)$ and $q_2(0)$, the following observations were made:

<table>
<thead>
<tr>
<th>$x(t)$</th>
<th>$q_1(0)$</th>
<th>$q_2(0)$</th>
<th>$y(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>$e^{-t}tu(t)$</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>$e^{-t}(3t+2)u(t)$</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>-1</td>
<td>-1</td>
<td>$2u(t)$</td>
</tr>
</tbody>
</table>

Determine $y(t)$ when both the initial conditions are zero and the input $x(t)$ is as shown in Fig. 1.7-11. [Hint: There are three causes: the input and each of the two initial conditions. Because of the linearity property, if a cause is increased by a factor $k$, the response to that cause also increases by the same factor $k$. Moreover, if causes are added, the corresponding responses add.]