I. Probability & Set Theory

- many situations occur in which it is impossible to exactly replicate what is important --> probability
- 3 major aims of this course:
  - logic behind probability theory
  - develop intuition into applying the theory
  - applications to engineering problems
  - i.e., critical and mathematical skills

Probability & Set Theory

- OUTCOME of an experiment: any possible observation
- SAMPLE SPACE: set of all possible outcomes
- An EVENT: some set of outcomes
  - EVENT SPACE: set of events
Probability Axioms

- probability measure $P[.]$ maps Sample Space $\rightarrow$ Real interval $[0, 1]$
- $P[A] > 0$, $P[S] = 1$,
- for mutually exclusive events
  \[ P[A \cup B \cup C \cup \ldots] = P[A] + P[B] + P[C] + \ldots \]

Conditional Probability

- law of total probability
- Bayes’ theorem

Statistical Independence

- 2 events $A$, $B$ are independent iff
  \[ P[AB] = P[A] \cdot P[B] \]
- extend to 3 or more events
Sequential Experiments, Tree Diagrams

- Bayes’ theorem in tree structure
  - at each branch level, the probability sum = 1

Counting

- if expt. A has $n$ outcomes, and expt B has $k$ outcomes, then performing expts A and B yields $nk$ outcomes
- $n$ distinguishable objects:
  - total number of $k$-permutations = \( \frac{n!}{(n-k)!} \)
  - total number of ways to choose $k$ objects
    \[ \binom{n}{k} = \frac{n!}{k!(n-k)!} \]

Independent Trials

- In $n$ indep. Trials: for $k$ successes
  - each outcome has probability \( p^k (1-p)^{n-k} \)
  - there are \( \frac{n!}{k!(n-k)!} \) such outcomes
- reliability
- multiple outcomes