Revised: August 22, 2025

ECE 601 Linear Systems

Practice Problems Set #1

Due Date: Not collected. Solutions posted by September 21, 2025.

1. **Linearization of State Space Model**: an n-dimensional state space system

$$\dot{x} = F(x, u), \quad x(0) \text{ given} \tag{1a}$$
$$y = H(x, u) \tag{1b}$$

with an equilibrium at (x_e, u_e). A linear state space model (A, B, C, D) for the system (F, H) is generally computed from the Taylor series of F and H about an equilibrium.

(a) Derive an explicit formula for the linear model in terms of (F, H) and (x_e, u_e).

(b) Apply your results from part (a) to the spring-electromagnet system in Homework #2 for an arbitrary set of parameters m, g, k_s, d_s, k_m, d_m and an arbitrary equilibrium (x_e, u_e).

2. **Numerically Solving Hilbert Systems**: A Hilbert system is a set of linear equations $Hx = b$, where H is the Hilbert matrix (this means: $H_{ij} = 1/(i+j-1)$ for all $i, j \geq 1$). For Hilbert matrices of dimension $n = 5, 7, 9, \ldots, 15$, try the following experiment:

i. Compute H^{-1} using the MatLab command `inv` and save.

ii. Set $b = [1, 2, \ldots, n]^T$.

iii. Compute $x = H^{-1}b$.

iv. Compute $bb := Hx$.

(a) For each value of n compare b and bb. Do you get what you expect ? How big of a Hilbert system can you solve accurately?

(b) Plot $\log(1/c(H))$ as a function of n and compare these values against the value of $\log(eps)$ for your MatLab installation. (The MatLab parameter *eps* can differ from machine to machine.) Can you relate this data to your observations in part (a)? Explain.

3. **Properties of Condition Numbers**: Let $A, B \in \mathbb{R}^{n \times n}$ be arbitrary matrices. (Assume A is invertible when necessary.) For each statement below, either prove its validity *in general* or provide a *specific* counterexample to disprove it.

(a) $c(A) \geq 1$

(b) If $A^{-1} = A^T$ then $c(A) = 1$

(c) If $c(A) = 1$ then $A^{-1} = A^T$

(d) $c(A^T) = c(A)$

(e) $c(A^{-1}) = (c(A))^{-1}$

(f) $c(\alpha A) = \alpha c(A)$, $\forall \alpha \in \mathbb{R}$

(g) $c(A + B) \leq c(A) + c(B)$

(h) $c(AB) \leq c(A)c(B)$