

ECE 601 Midterm Exam 2, Fall 2025

See [exam announcement](#) for full exam instructions and rules.

Exams must arrive at sgray@odu.edu by 7:20 pm EST, October 30, 2025.

Show all work and simplify all answers for full credit.

1. **Nonlinear I/O ODE:** An input-output system $u \mapsto y$ is described by the nonlinear ODE

$$\ddot{y} + 3\dot{y} + y^2 = u.$$

- (a) {15 points} Linearize the equation about an equilibrium (u_e, y_e) and determine the corresponding transfer function when $u_e = y_e > 0$.
 - (b) {10 points} Compute the first five Markov parameters of the linearized model in part (a)
 - (c) {10 points} Determine the rank of the largest system Hankel matrix one can construct with the Markov parameters found in part (b). Does this result make sense? Explain.
2. **Perfect Output Conditioning:** Let (A, b, c) be a state space realization of a transfer function $H(s)$

- (a) {15 points} Determine sufficient conditions under which a state space transformation T exist so that the new realization has an observability matrix with condition number equal to one?
- (b) {10 points} Suppose $H(s)$ has a state space realization

$$A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad c = [1 \quad 2].$$

If possible, determine T as described in part (a) for this system and verify that it works as expected. If not possible, explain why not.

- (c) {10 points} When such a T exists, is it unique? If yes, explain why. If no, provide another T with this property.
3. **Discrete-time System:** Consider the discrete-time system

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 4 & -10 \\ 3 & -7 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} -6 & 10 \end{bmatrix} x(k). \end{aligned}$$

- (a) {5 points} Compute the transfer function $H(z)$ for this system.
- (b) {5 points} Is the system reachable?
- (c) {5 points} Is the system controllable?
- (d) {5 points} Is the system observable?
- (e) {5 points} Is the system constructible?
- (f) {5 points} Is the system minimal?

Remark: Explicitly justify all answers for full credit.