ECE 601 Homework #8 Solutions

1. Kailath Problem 3.1-1

Clearly

$$Y(s) = \frac{H(s)G_c(s)}{1+H(s)G_c(s)}$$

$$= \frac{-(s-0.5)K_2(s+\delta)}{s(s-1)(s+\gamma)-K_2(s-0.5)(s+\delta)}$$

$$\Rightarrow \alpha(s) = s^3+(\gamma-K_2-1)s^2+(0.5K_2-K_2\delta-\gamma)+0.5K_2\delta$$

$$= s^3+3s^2+3s+1$$

Equate coefficients

$$\gamma-K_2-1=3$$

$$0.5K_2-K_2\delta-\gamma=3$$

$$0.5K_2\delta=1 \rightarrow K_2\delta=2$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 0.5 \end{bmatrix}\begin{bmatrix} \gamma \\ K_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \boxed{K_2=-18,\ \gamma=-14,\ \delta=-1/9}$$

2. Kailath Problem 3.2-3

Let $b(s)/a(s)$ be the open-loop transfer function, and $b(s)/\alpha(s)$ be the closed-loop transfer function. Since state feedback preserves system order then

$$\deg(\alpha(s)) = \deg(a(s)).$$

In which case,

$$\begin{aligned} r_{CL} &\triangleq \deg(\alpha(s)) - \deg(b(s)) \\ &= \deg(a(s)) - \deg(b(s)) \\ &= r_{OL}. \end{aligned}$$

So the relative degree in invariant with respect to (constant) state feedback.

3. (a) From Homework #5, the open-loop system has transfer function

$$H(s) = \frac{\beta}{s^2 - \alpha},$$

where $\alpha < 0$, $\beta > 0$. The closed-loop system has the transfer function

$$H_{cl}(s) = \frac{H(s)}{1 + kH(s)}$$

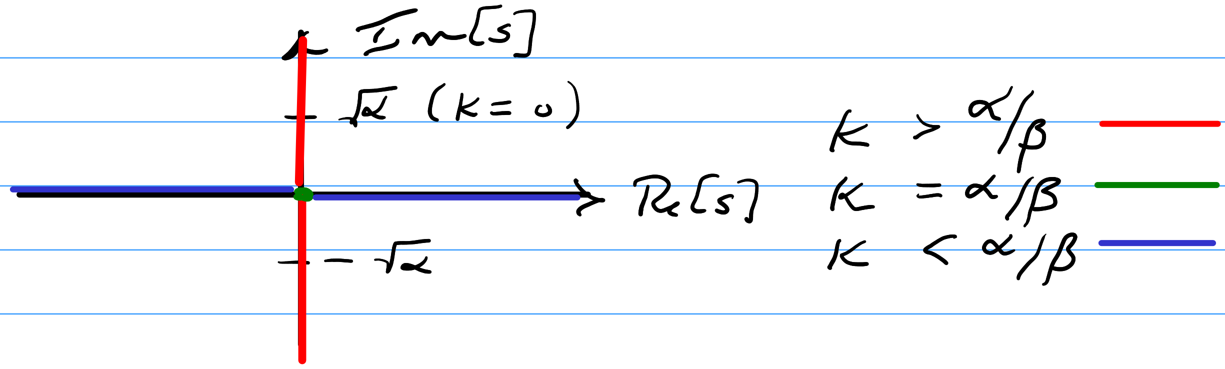
$$= \frac{\beta}{s^2 - \alpha + k\beta}.$$

So the closed-loop modes are

$$s_\pm = \pm(\alpha - k\beta)^{1/2}.$$

gain	CL roots
$k > \alpha/\beta$	complex roots
$k = \alpha/\beta$	roots at $s = 0$
$k < \alpha/\beta$	real roots

The situation is summarized in the following root locus diagram.



So output feedback cannot asymptotically stabilize this system.

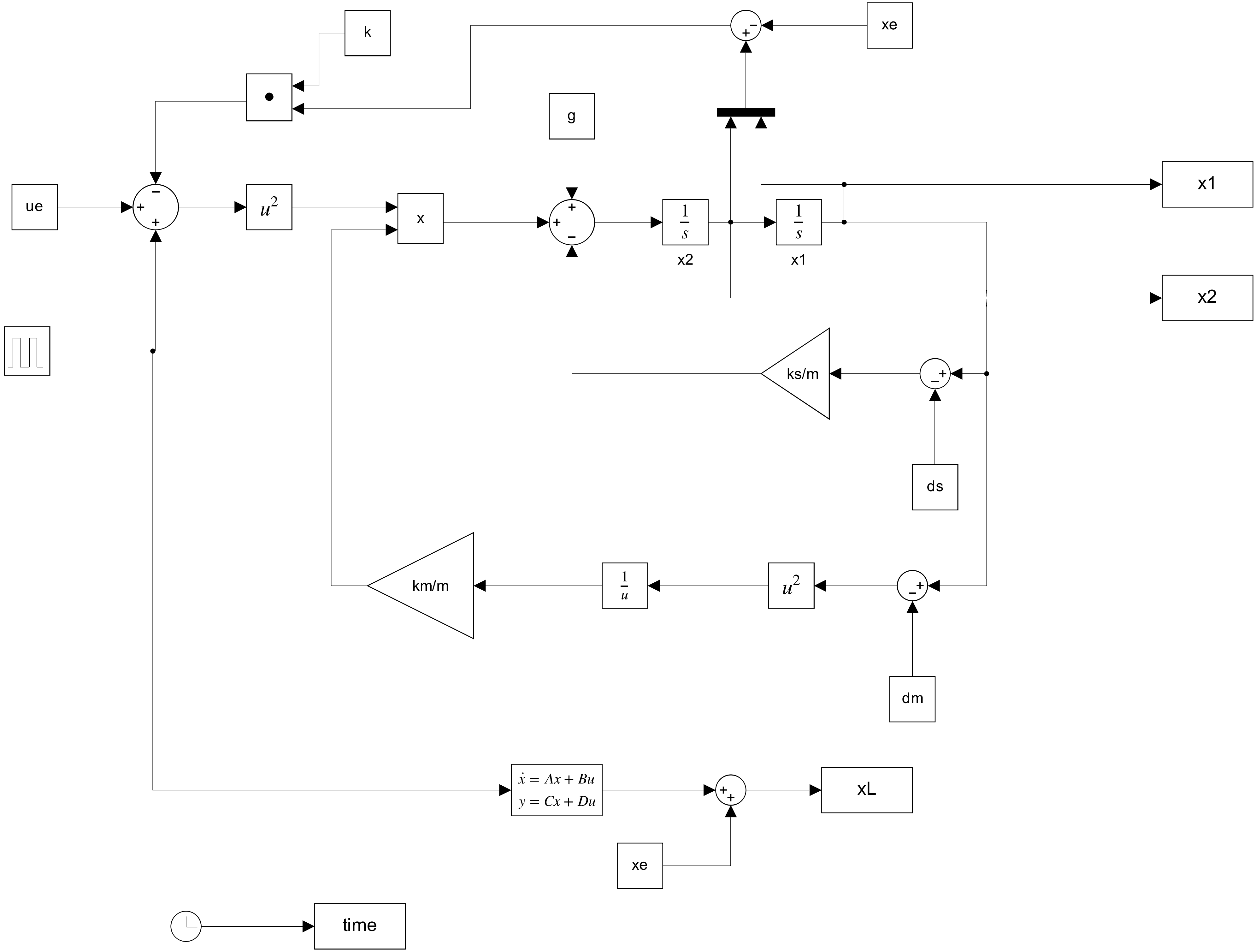
(b) Since it was shown in Homework #6 that the system is controllable, a state feedback controller will work. The desired pole locations were selected to add damping without changing the natural frequency of the system. See the attached code for the design details.

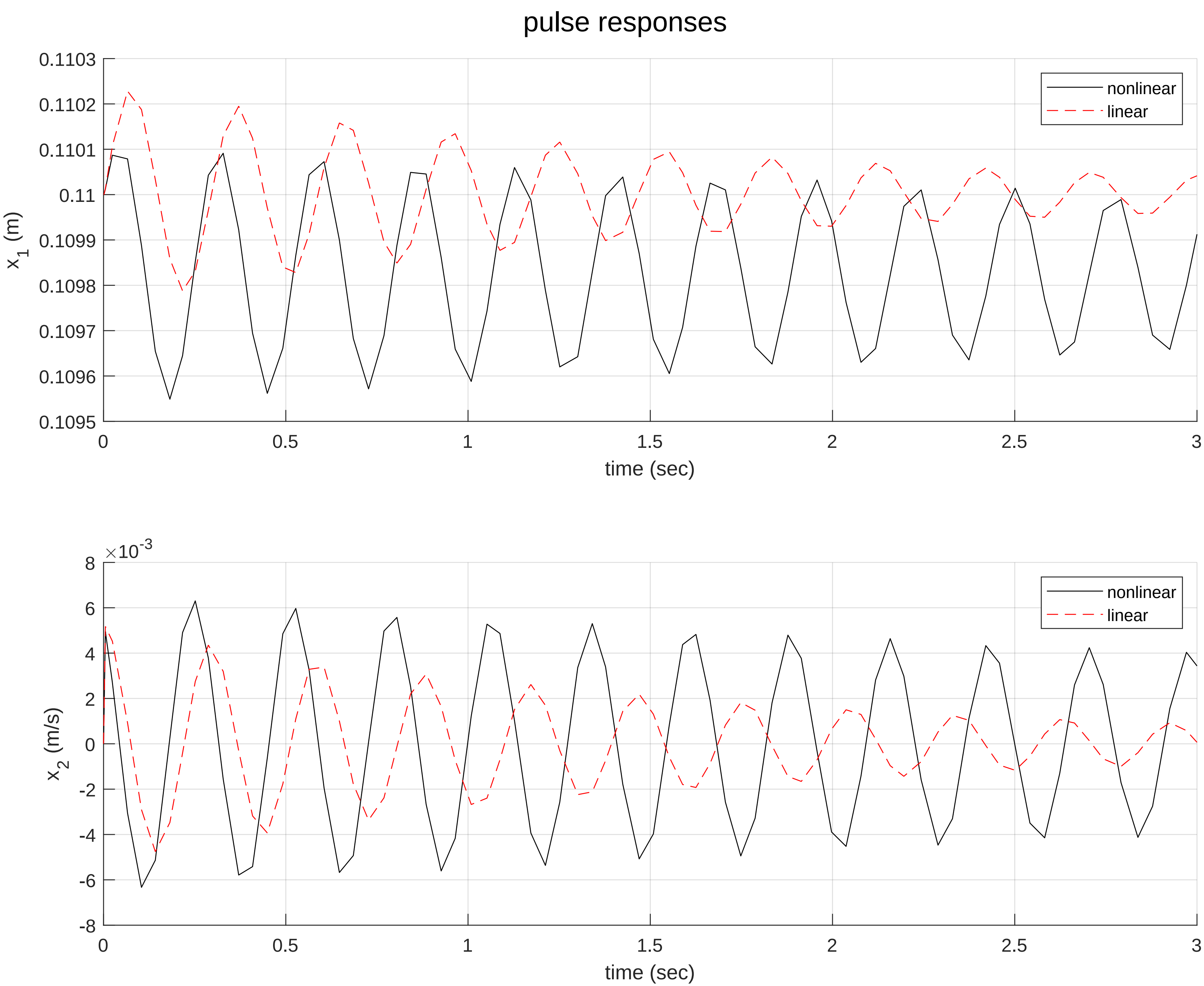
(c) See the attached plots. The same pulse as in Homework #4 was applied. The linear system is now asymptotically stable.

(d) The nonlinear system is also asymptotically stable, but observe that the linear model is overly optimistic about the effects of the damping.

```
%
% ECE 601 Fall 2025
%
% Homework 8, Problem 3
%
% MATLAB R2023a
%

clc;
clear all;
close all;

% system parameters

m=0.01;
g=9.8;
ks=15;
ds=0.10;
km=0.005;
dm=0.12;

% equilibrium function

x1e=0.11;
x2e=0;
xe=[x1e x2e]';
ue=sqrt(((ks/m)*(x1e-ds)-g)*(m/km)*(x1e-dm)^2);

% linear model

alpha=-(ks/m)-(2*km/m)*(((ue^2)/((x1e-dm)^3)));
beta=(2*km/m)*(ue/((x1e-dm)^2));
A=[0 1;alpha 0];
b=[0 beta]';
c=eye(2);
d=zeros(2,1);
ltiSys = ss(A,b,c,d);
sys = tf(ltiSys);

% state feedback

Im_modes=imag(eig(A));
omega=Im_modes(1);
period=(2*pi)/omega; % open-loop systems period
sigma=2; % damping
clpoles=[-1*sigma*period+1i*omega -1*sigma*period-1i*omega];
k=place(A,b,clpoles);
Acl=A-b*k;
```

```
% Run the Simulink simulation

T=3;
pulseon=0.0;
pulsewidth=0.05;
pulsesize=0.1*ue;

sim('spring_electromagnet_system',T);

figure(1)

subplot(2,1,1);
hold on;
grid on;
plot(time,x1,'k-');
plot(time,xL(:,1),'r--')
legend('nonlinear','linear')
xlabel('time (sec)');
ylabel('x_1 (m)');
hold off

subplot(2,1,2);
hold on;
grid on;
plot(time,x2,'k-');
plot(time,xL(:,2),'r--')
legend('nonlinear','linear')
xlabel('time (sec)');
ylabel('x_2 (m/s)');
sgtitle('pulse responses');
hold off;

orient 'landscape';
print('hw8 prob3','-dpdf','-fillpage');
```


k
xe
g
ue
u^2
x
$\frac{1}{s}$
x2
$\frac{1}{s}$
x1
x1
x2
ks/m
ds
km/m
$\frac{1}{u}$
u^2
dm
$\dot{x} = Ax + Bu$
$y = Cx + Du$
xe
xL
time


pulse responses
0.1103
0.1102
0.1101
0.11
0.1099
0.1098
0.1097
0.1096
0.1095
x1 (m)
0
0.5
1
1.5
2
2.5
3
time (sec)
nonlinear
linear
×10-3
8
6
4
2
0
-2
-4
-6
-8
x2 (m/s)
0
0.5
1
1.5
2
2.5
3
time (sec)
nonlinear
linear