ECE 601 Homework #7 Solutions

1. Kailath Problem 2.6-3: Solution is in the text.

2. Kailath Problem 2.6-4

(a)

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$e = -y$$

$$u = -y + y^3 = -x_1 + x_1^3$$

$$\dot{x}_1 = -x_1 + x_2 + x_1^3$$

$$\dot{x}_2 = -x_1 + x_1^3$$

(b) $V(x) = x_1^2 + x_2^2$

$$\dot{V}(x) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2$$

$$= 2x_1(-x_1 + x_2 + x_1^3) + 2x_2(-x_1 + x_1^3)$$

$$= -2x_1^2(1 - x_1^2 - x_1 x_2)$$

< 0 if $\|x\|$ is small.

3. See the attached MATLAB code.

(a)

zeros= -1.3333

0.5000

poles= -0.5000

0

0.5000

modes = -0.5000

0

0.5000

CT case: NOT BIBO stable and NOT internally asymptotically stable.

DT case: BIBO stable and internally asymptotically stable.

(b)

zeros = -2

6

poles = -2.0000

0

-0.7500

modes = -2.0000

0

-0.7500

CT case: NOT BIBO stable and NOT internally asymptotically stable.

DT case: BIBO stable but NOT internally asymptotically stable.

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%
% ECE 601 Homework #7, Problem 3 Fall 2025
%

clear all;

% part(a)
A=[2.5 0 3; -2.5 2.5 -2.5; -4.5 2 -5];
b=[1 1 0]';
c=[0 3 -1];
d=[0];
sys=ss(A,b,c,d);
zero(sys)
pole(sys)
eig(A)

% part(b)
A=[0.5 -0.5 2.5; -1.25 1.25 -1.25; -2.5 2.5 -4.5];
b=[-2 1 2]';
c=[2 1 1];
d=[0];
sys=ss(A,b,c,d);
zero(sys)
pole(sys)
eig(A)
```

4. (a) $$V(z) = (T^T z)^T P (T^T z) = z^T (TPT^T) z$$
$$= z^T \Sigma z$$
$$= \sum_{i=1}^{n} \lambda_i(P) z_i^2$$

(b) Clearly,

$V(\cancel{x}z)$ is positive definite $\iff \lambda_i(P) > 0 \ \forall i$
$V(\cancel{x}z)$ is positive semi-definite $\iff \lambda_i(P) \geq 0 \ \forall i$
$V(\cancel{x}z)$ is negative definite $\iff \lambda_i(P) < 0 \ \forall i$
$V(\cancel{x}z)$ is negative semi-definite $\iff \lambda_i(P) \leq 0 \ \forall i$

(c) $V(x)$ is positive definite, for example, if and only if

$$V(x) > 0 \quad \forall x \in \mathbb{R}^n - \{0\}. \qquad (1)$$

Since $z = Tx$ is a one-to-one map on $\mathbb{R}^n$, then (1) holds if and only if

$$V(z) > 0 \quad \forall z \in \mathbb{R}^n - \{0\}.$$

Therefore $V(x)$ is positive definite if and only if $V(z)$ is positive definite.

(d) Combining the previous results, $V(x)$ is positive definite if and only if $\lambda_i(P) > 0$, $i = 1, 2, \ldots n$. Likewise for the other forms of definiteness.