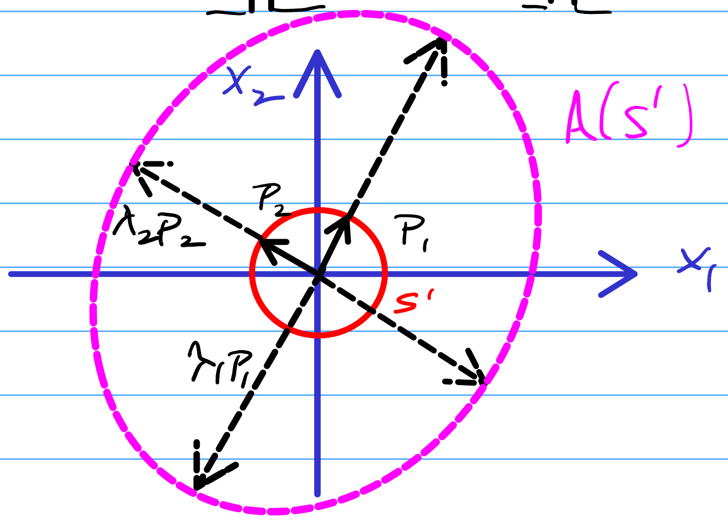
ECE 601 Homework #3 Solutions

1. (a) The spectral decomposition of A is

$$A = \underbrace{\begin{bmatrix} 0.5257 & -0.8507 \\ 0.8507 & 0.5257 \end{bmatrix}}_{P_1 \quad P_2} \begin{bmatrix} -3.8541 & 0 \\ 0 & 2.8541 \end{bmatrix}_{\lambda_1,\ \lambda_2} \begin{bmatrix} 0.5257 & -0.8507 \\ 0.8507 & 0.5257 \end{bmatrix}^T.$$



(b) Since $A = A^T$, $c(A) = |\lambda_{max}(A)| / |\lambda_{min}(A)| = \underline{1.3504}$.

(c) If $x \in A(S')$ then there exists an $s \in S'$ such that $x = As$. Hence,

$$\|A^{-1}x\|^2 = \|s\| = 1$$

$$x^T A^{-T} A^{-1} x = \underbrace{x^T P}_{\tilde{x}^T} \Sigma^{-2} \underbrace{P^T x}_{\tilde{x}} = 1$$

$$\frac{\tilde{x}_1^2}{\lambda_1^2} + \frac{\tilde{x}_2^2}{\lambda_2^2} = 1$$

$\tilde{x}$ is a rotation of x by an angle θ (see the figure above).

Then $\tilde{x}$ satisfies the equation of an ellipse.

(d) The logic we saw in lecture implies that the first equality follows from setting

$$\Delta b = P_1, \quad b = P_2.$$

A "reverse" argument suggests the second equality comes from setting

$$\Delta b = P_2, \quad b = P_1.$$

2. (a) Check the properties:

i, Since $\|Ax\| \geq 0$ and $\|x\| > 0$ then $\max_{x \neq 0} \frac{\|Ax\|}{\|x\|} \geq 0$ ✓

ii, If $A = 0$ then $\|Ax\| = 0 \ \forall x$ or $\|A\| = 0$. ✓
If $\|A\| = 0$ then $\max_{x \neq 0} \|Ax\|/\|x\| = 0 \Rightarrow Ax = 0 \ \forall x$
$\Rightarrow A = 0$. ✓

iii, $$\|\alpha A\| = \max_{x \neq 0} \frac{\|\alpha A x\|}{\|x\|} = \max_{x \neq 0} \frac{|\alpha| \|Ax\|}{\|x\|}$$

$$= |\alpha| \|A\|. ✓$$

iv, $$\|A+B\| = \max_{x \neq 0} \frac{\|(A+B)x\|}{\|x\|} \leq \max_{x \neq 0} \frac{\|Ax\| + \|Bx\|}{\|x\|}$$

$$\leq \|A\| + \|B\|. ✓$$

(b) $\|A\| = |\lambda_1|$

(c) $$\|AB\| = \max_{x \neq 0} \frac{\|A(Bx)\|}{\|x\|} = \max_{x \neq 0} \frac{\|A(Bx)\|}{\|Bx\|} \cdot \frac{\|Bx\|}{\|x\|}$$

$$\leq \max_{y \neq 0} \frac{\|Ay\|}{\|y\|} \cdot \max_{x \neq 0} \frac{\|Bx\|}{\|x\|}$$

$$= \|A\| \cdot \|B\|$$ ↘

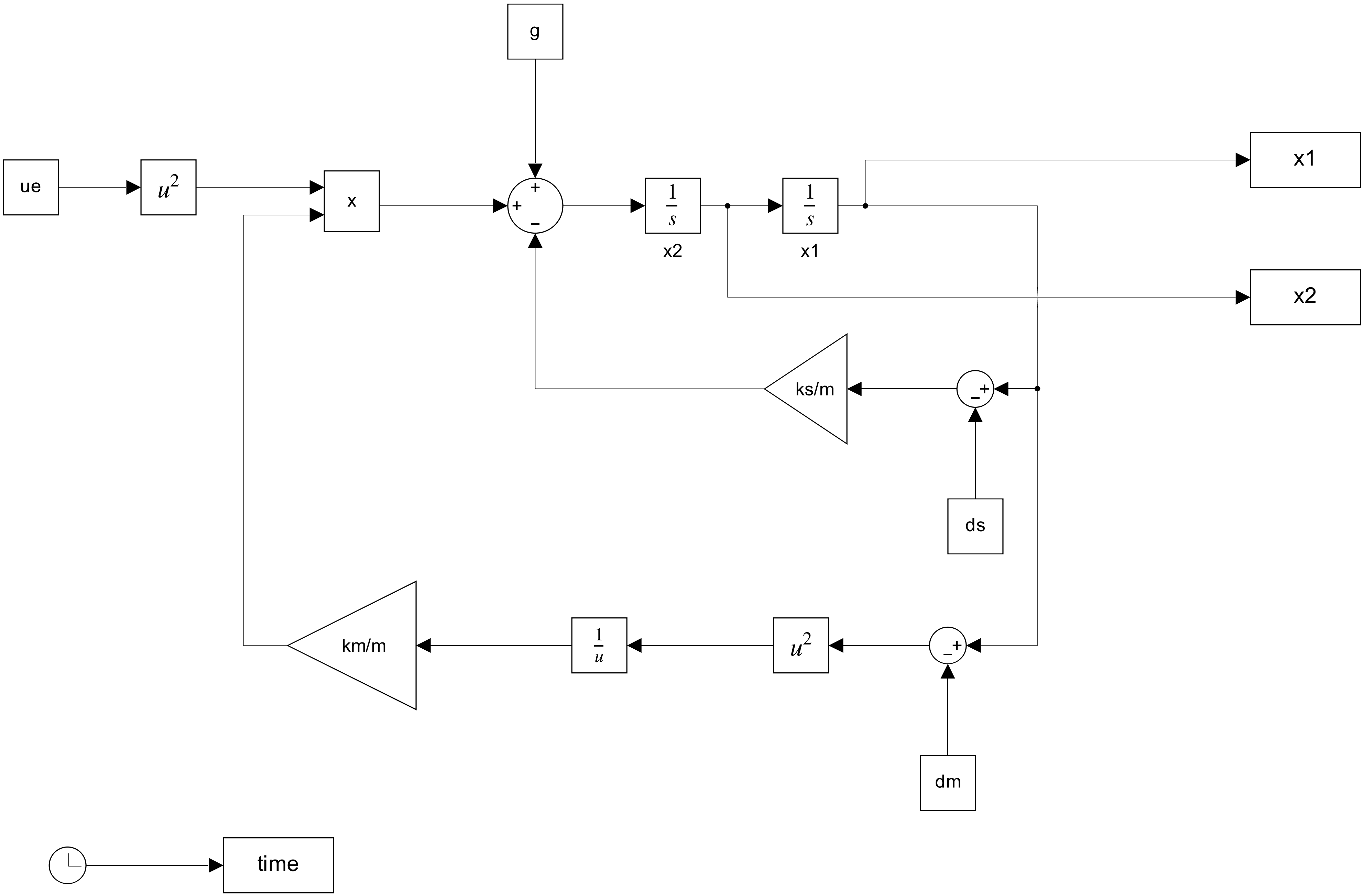
3. See the attached Matlab code, Simulink diagram, and plots.

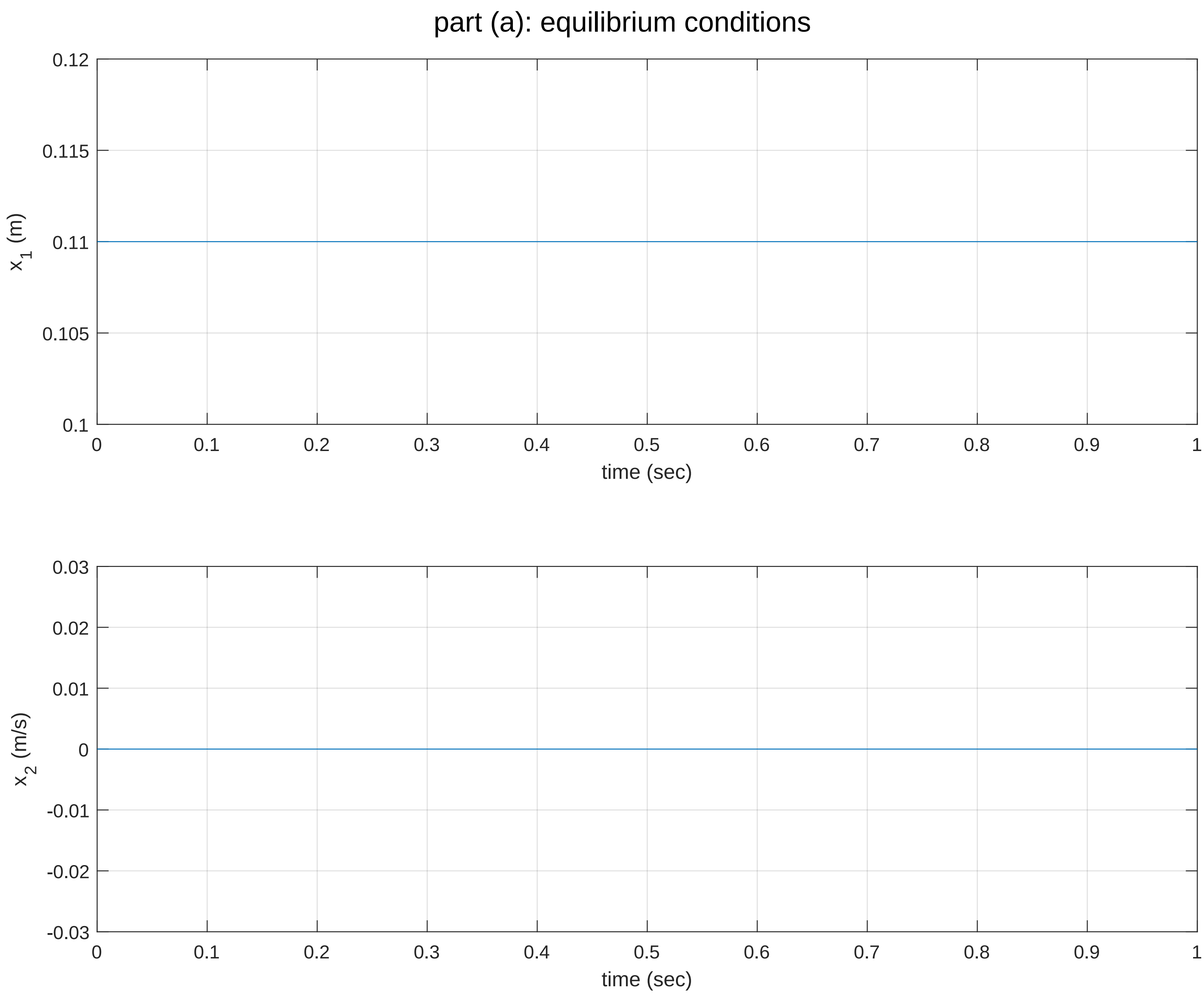
(a) If the system is initialized in equilibrium, then all the states should be constant as shown in the first figure.

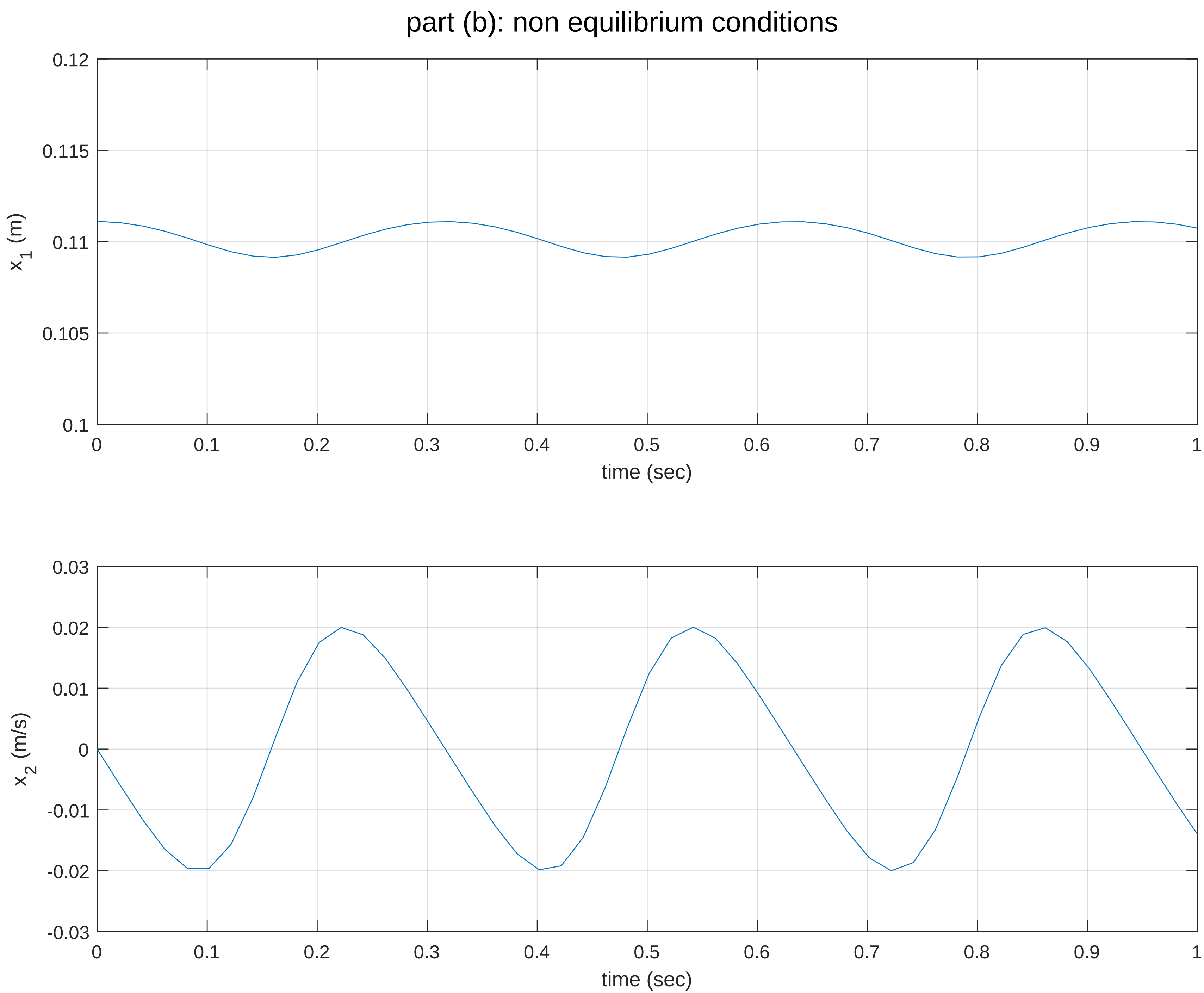
(b) For any other initial condition, the states will Not be constant as shown in the second figure. We see instead an oscillation.

```
%
% ECE 601 Fall 2025
%
% Homework 3, Problem 3
%
% MATLAB R2023a
%

clc;
clear all;
close all;

% system parameters

m=0.01;
g=9.8;
ks=15;
ds=0.10;
km=0.005;
dm=0.12;

% equilibrium function

x1e=0.11;
x2e=0;
ue=sqrt(((ks/m)*(x1e-ds)-g)*(m/km)*(x1e-dm)^2)

%%%% part (a)

% Run the Simulink simulation
T=1;
sim('spring_electromagnet_system',T);

figure(1)
hold on;
grid on;

subplot(2,1,1);
plot(time,x1);
grid on;
xlabel('time (sec)');
ylabel('x_1 (m)');
axis([0,T,0.10,0.12]);

subplot(2,1,2);
plot(time,x2);
grid on;
xlabel('time (sec)');
ylabel('x_2 (m/s)');
```

```
axis([0,T,-0.03,0.03]);

sgtitle('part (a): equilibrium conditions');

hold off;

orient 'landscape';
print('hw3 prob3 plot1','-dpdf','-fillpage');

%%%% part (b)

x1e=x1e*1.01; % 1 percent change in initial position

% Run the Simulink simulation

sim('spring_electromagnet_system',T);

figure(2)
hold on;
grid on;

subplot(2,1,1);
plot(time,x1);
grid on;
xlabel('time (sec)');
ylabel('x_1 (m)');
axis([0,T,0.10,0.12]);

subplot(2,1,2);
plot(time,x2);
grid on;
xlabel('time (sec)');
ylabel('x_2 (m/s)');
axis([0,T,-0.03,0.03]);

sgtitle('part (b): non equilibrium conditions');

hold off;

orient 'landscape';
print('hw3 prob3 plot2','-dpdf','-fillpage');
```


g
ue
u^2
x
+
+
−
$\frac{1}{s}$
$\frac{1}{s}$
x2
x1
x1
x2
ks/m
+
−
ds
km/m
$\frac{1}{u}$
u^2
+
−
dm
time


part (a): equilibrium conditions
0.12
0.115
0.11
0.105
0.1
x_1 (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
time (sec)
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
x_2 (m/s)
time (sec)



part (b): non equilibrium conditions
0.12
0.115
0.11
0.105
0.1
x_1 (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
time (sec)
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
x_2 (m/s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
time (sec)