ECE 601 Linear Systems

Homework #3

Due Date: September 18, 2025

1. **Condition Numbers and Error Gains**: Consider a linear system of equations $Ax = b$, where

$$A = \begin{bmatrix} 1 & -3 \\ -3 & -2 \end{bmatrix}.$$

   (a) Sketch the set $A(S^1)$, namely the image of the unit circle in $\mathbb{R}^2$ under A.

   (b) Compute $c(A)$ and compare it to the sketch in part (a).

   (c) Derive the mathematical equation that a point x in $A(S^1)$ must satisfy.

   (d) For each equation below, determine specific vectors b and Δb that satisfy the equality.

$$\begin{aligned} \frac{\|\Delta x\|}{\|x\|} &= c(A)\frac{\|\Delta b\|}{\|b\|} \\ \frac{\|\Delta x\|}{\|x\|} &= \frac{1}{c(A)}\frac{\|\Delta b\|}{\|b\|}. \end{aligned}$$

2. **Normed Vector Spaces**: A vector space V is called a *normed vector space* if every vector in the space can be assigned a *length* in a natural way. Specifically, there must exist a function $\|\cdot\| : V \mapsto \mathbb{R}$ such that

   i. $\|x\| \geq 0, \forall x \in V$

   ii. $\|x\| = 0$ if and only if $x = 0$

   iii. $\|\alpha x\| = |\alpha|\|x\|$, $x \in V$ and $\alpha \in \mathbb{R}$.

   iv. $\|x + y\| \leq \|x\| + \|y\|, \forall x, y \in V$.

   (a) From Homework #1 we know that the set of all $n \times n$ matrices, $\mathbb{R}^{n\times n}$, forms a vector space. Show that the function

$$\begin{aligned} \|\cdot\| &: \mathbb{R}^{n\times n} \mapsto \mathbb{R} \\ \|\cdot\| &: A \mapsto \|A\| = \max_{x\neq 0} \frac{\|Ax\|}{\|x\|} \end{aligned}$$

   satisfies all the necessary properties of a norm on this space.

   (b) Determine the norm of matrix A given in Problem 1 above.

   (c) Show that the norm defined in part (a) satisfies the submultiplicative property $\|AB\| \leq \|A\| \cdot \|B\|$.

3. **Simulation of Spring-Electromagnet System in Equilibrium**: Build a Simulink simulation of the spring-electromagnet system using your state space system from Homework #2. (Not familiar with Simulink? Go here).

   (a) Initialize your system in the equilibrium condition corresponding to $x_{1e} = 0.11$ m and plot the simulated state responses. What do you expect from theory, and what did you observe? Explain any differences.

   (b) Repeat part (a) but now with a small perturbation added to the initial state.