ECE 601 Homework #2 Solutions

1. (a) The pair (x_e, u_e) must satisfy the algebraic equation

$$\dot{x} = 0 = F(x_e, u_e). \qquad (1)$$

(b) Since F is often not linear, (1) can have more than one solution. Usually, (1) has to be solved numerically.

2. (a) $m\ddot{d} = F_s + F_m + F_g$

$$= -k_s(d - d_s) + K_m \frac{i^2}{(d-d_m)^2} + mg.$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_s}{m}(x_1 - d_s) + \frac{k_m}{m}\frac{u^2}{(x_1 - d_m)^2} + g$$

$$\boxed{\begin{aligned} F(x,u) &= \begin{bmatrix} x_2 \\ -\frac{k_s}{m}(x_1 - d_s) + \frac{k_m}{m}\frac{u^2}{(x_1-d_m)^2} + g \end{bmatrix} \\ H(x,u) &= x_1 \end{aligned}}$$

(b) If $\dot{x} = 0$ then $\dot{x}_2 = 0$ and

$$-\frac{k_s}{m}(x_{1e} - d_s) + \frac{k_m}{m}\frac{u_e^2}{(x_{1e} - d_m)^2} + g = 0$$

So every pair (x_e, u_e) that satisfies the equation above is an equilibrium.

(c) If x_{1e} is fixed, then

$$u_e^2 = \left(\frac{k_s}{m}(x_{1e} - d_s) - g\right)\frac{m(x_{1e} - d_m)^2}{k_m}.$$

For the parameter values given $u_e = 0.0322$ A.

3. (a) Define for $a, b \in S$ and $c \in \mathbb{R}$

$$ca = (ca_1, ca_2, \dots)$$
$$a + b = (a_1 + b_1, a_2 + b_2, \dots).$$

This gives a vector space structure.

(b) Observe

$$A(ca) = (ca_1 + ca_2, ca_2 + ca_3, \dots) = cA(a) \checkmark$$
$$A(a+b) = (a_1 + b_1 + a_2 + b_2, a_2 + b_2 + a_3 + b_3, \dots)$$
$$= A(a) + A(b) \checkmark$$

So A is linear.

(c) If (λ, p) is an eigenpair then

$$A(p) = (p_1+p_2, p_2+p_3, \ldots) = \lambda(p_1, p_2, \ldots)$$

So

$$\begin{aligned} \lambda p_1 &= p_1 + p_2 \longrightarrow p_2 = (\lambda-1)p_1 \\ \lambda p_2 &= p_2 + p_3 \longrightarrow p_3 = (\lambda-1)p_2 = (\lambda-1)^2 p_1 \\ &\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \end{aligned}$$

Thus, for <u>any</u> $\lambda \in \mathbb{R}$

$$p = p_1\,(1, (\lambda-1), (\lambda-1)^2, \ldots), \quad p_1 \in \mathbb{R} - \{0\}$$

is an eigenvector.