ECE 601 Linear Systems

Homework #2

Due Date: September 11, 2025

1. **System Equilibria**: Consider an n-dimensional state space system

$$\dot{x} = F(x,u), \quad x(0) \text{ given} \tag{1a}$$
$$y = H(x,u). \tag{1b}$$

A fixed $x_e \in \mathbb{R}^n$ and constant input $u_e \in \mathbb{R}$ are called an *equilibrium state* and *equilibrium input*, respectively, if $x(t) = x_e$ for all $t > 0$ whenever one sets $x(0) = x_e$ and $u(t) = u_e$ is applied.

(a) Describe how to compute an equilibrium (x_e, u_e) in general.

(b) Are equilibria (x_e, u_e) unique? Explain.

2. **Equilibria of the Spring-Electromagnet System**: Reconsider the spring-electromagnet system in Homework #1.

(a) Derive a state space model of the form given in (1) with states $x_1 = d$ and $x_2 = \dot{d}$.

(b) Determine all the equilibria for this system.

(c) Now suppose that $m = 0.01$ Kg, $g = 9.8$ m/s, $k_s = 15$ N/m, $d_s = 0.1$ m, $k_m = 0.005$ N m^2/A^2, and $d_m = 0.12$ m. Compute u_e so that $x_{1e} = 0.11$ m.

Remark: The two problems above do **not** involve linearization in any way.

3. **Real Sequence Space**: Let S denote the set of all infinite real sequences. That is, every element of S looks like $a = (a_1, a_2, \dots)$, where $a_i \in \mathbb{R}$ for each $i \geq 1$.

(a) Does S have a vector space structure available? Explain in detail.

(b) Consider the operator

$$A : S \to S : (a_1, a_2, \ldots) \mapsto (a_1 + a_2, a_2 + a_3, \ldots).$$

Is A a *linear* operator on S? Explain.

(c) Determine any eigenvalue/eigenvector pairs that A might have.