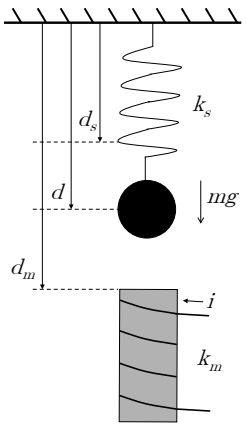
Revised: August 8, 2025

ECE 601 Linear Systems

Homework #1

Due Date: September 4, 2025

1. **Spring-Electromagnet System**: Consider the spring-electromagnet system shown below. The spring exerts a force $F_s = -k_s(d - d_s)$ on a metal ball of mass m, where k_s is the spring constant, d is the vertical position of the ball, and d_s is the equilibrium position of the spring when all other external forces are absent. The electromagnet applies a force $F_m = k_m i^2/(d - d_m)^2$, where k_m is the magnetic force constant, i is the coil current, and d_m is the location of the magnet's face. The ball also experiences a gravitational force $F_g = mg$.



   (a) If the input u is taken to be the coil current i and the output y is the ball's vertical position d, determine the input-output equation for this system.

   (b) What type of input-output equation describes this system? Differential or algebraic? Linear or nonlinear? Explain in as much detail as you can.

2. **Examples of Vector Spaces**: Many different types of mathematical objects can be used to form a vector space. For each set described below, try to impose a vector space structure on it by *explicitly* defining a notion of vector addition and scalar multiplication. Make sure that your definitions are *algebraically closed*, for example, if you add two elements in the set you must get another element in the set. Whenever a vector space structure is not possible, explain why not.

   (a) The set of all $n \times \ell$ matrices with real components, $\mathbb{R}^{n\times\ell}$.

   (b) The subset of all invertible matrices in $\mathbb{R}^{n\times n}$, $GL_n(\mathbb{R})$.

   (c) The set of all rational functions in the variable s, $\text{Rat}(s)$.

(d) The subset of all proper rational functions in $\mathrm{Rat}(s)$, $\mathrm{Rat}_{\mathrm{p}}(s)$.

3. **MatLab Introduction**: Get access to the MatLab software package (or some equivalent). Learn how to use it. Then perform the following tasks:

   (a) Produce a 10×10 *Hankel* matrix, H, whose first column is $[1\ 2\ \cdots\ 10]$ and whose last row is $[10\ 1\ 2\ \cdots\ 9]$. What is its basic structure? (*Hint*: For MatLab users type: `help hankel`).

   (b) Produce a 10×10 *Toeplitz* matrix, T, whose first column is $[1\ 10\ 9\ \cdots\ 2]$ and whose first row is $[1\ 2\ \cdots\ 10]$. How does the structure of T compare to that of H in part (a)? (*Hint*: `help toeplitz`).