

ECE 302 - Linear System Analysis

Practice Problems Set #2

Due Date: Not collected (Solutions posted by November 1, 2025.)

1. **Linear Transformations:** Determine which of the following mappings are linear transformations by explicitly checking whether superposition holds.

(a) $\mathcal{A} : \mathbb{R}^3 \rightarrow \mathbb{R}^3 : (x, y, z) \mapsto (y + 1, x - z, 2z)$

(b) $\mathcal{A} : \mathbb{R}^2 \rightarrow \mathbb{R}^3 : (x, y) \mapsto (x, y, y)$

2. **Matrix Representations:** Determine a matrix representation, if possible, for each mapping in Problem 1.
3. **Composing Linear Transformations:** Consider two linear transformations $\mathcal{A}$ and $\mathcal{B}$ with matrix representations

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix},$$

respectively. Find matrix representations for the following linear transformations:

(a) $\mathcal{A} \circ \mathcal{B}$

(b) $(\mathcal{A} \circ \mathcal{B})^{-1}$

(c) $\mathcal{B}^{-1} \circ \mathcal{A}^{-1}$.

4. **Eigen Analysis:** Find the eigenvalues and eigenvectors of the matrices A and B given in Problem 3.
5. **Matrix Exponential:** Using the A matrix in Problem 3, compute the following:

(a) e^{At}

(b) The eigenvalues of e^{At} .

(c) The eigenvectors of e^{At} .