ECE 302 – HW#6 SOLUTIONS FALL 2025

Pb#01 (6-3-6): ⓐ trigonometric Fourier Series $x(t)$.

from C_n plot:

at $\omega = 0 \Rightarrow C_0 = 2$

at $\omega = 2 \Rightarrow C_1 = 2$

at $\omega = 3 \Rightarrow C_2 = 1$

from θ_n plot:

at $\omega = 2 \Rightarrow \theta_2 =$

at $\omega = 3 \Rightarrow \theta_3 = -\pi/2$

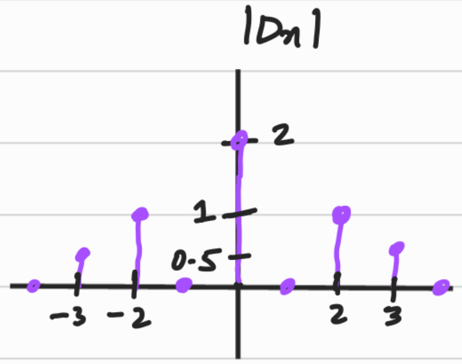
so,

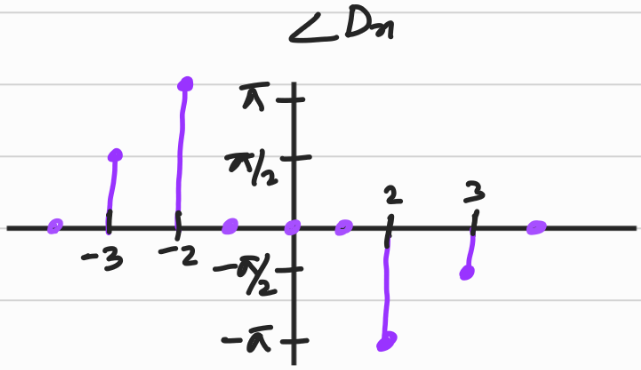
$$x(t) = C_0 + C_1 \cos(2t + \theta_2) + C_2 \cos(3t + \theta_3)$$

$$x(t) = 2 + 2\cos(2t - \pi) + \cos(3t - \pi/2)$$

$$\boxed{x(t) = 2 - 2\cos(2t) + \sin(3t)}$$

Pb#01 (6-3-6) ⓑ Sketch Exponential Fourier Spectra.





Pb#01 (6-3-6): ⓒ Exponential Fourier Series for $x(t)$

using sketches in part ⓑ:

$$\Rightarrow x(t) = \sum_{n=-\infty}^{\infty} |D_n| e^{\pm j(n\omega_0 t + \angle D_n)}$$

$$\Rightarrow x(t) = |D_0| + |D_2| e^{-j(2t-\pi)} + |D_{-2}| e^{j(2t-\pi)} +$$

$$|D_3| e^{-j(3t-\pi/2)} + |D_{-3}| e^{j(3t-\pi/2)}$$

$$\Rightarrow \boxed{x(t) = 2 + e^{j(2t-\pi)} + e^{-j(2t-\pi)} + \frac{1}{2} e^{j(3t-\pi/2)} + \frac{1}{2} e^{-j(3t-\pi/2)}}$$

Pb#01(6-3-6): d use $x(t)$ from c and apply:

Euler's Identity: $e^{-j\theta} + e^{j\theta} = 2\cos(\theta)$

$$\Rightarrow x(t) = 2 + 2\cos(2t-\pi) + \frac{1}{2}(2\cos(3t-\pi/2))$$

$$\Rightarrow x(t) = 2 + 2\cos(2t-\pi) + \cos(3t-\pi/2)$$ ✓ checked!

Pb#2(6.3-10): a Exponential Fourier Series for $\hat{x}(t) = x(t-T)$

since, $x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$

so,

$$\hat{x}(t) = x(t-T) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0(t-T)}$$

$$\hat{x}(t) = \sum_{n=-\infty}^{\infty} (D_n e^{-jn\omega_0 T}) e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \hat{D}_n e^{jn\omega_0 t}$$

where, $\hat{D}_n = D_n e^{-jn\omega_0 t} \Rightarrow |\hat{D}_n| = |D_n|$ and

$\angle \hat{D}_n = \angle D_n - jn\omega_0 T$

Showed!

Pb#03(6.4-5): a Exponential Fourier Series for periodic signal $x(t)$ (Fig P6.4-5 a)

from figure: $\omega_0 = 2\pi$ and,

$$D_n = \int_0^1 e^{-t} e^{-jn\omega_0 t}\, dt = \int_0^1 e^{(-1-jn\omega_0)t}\, dt$$

$$D_n = \left. \frac{e^{(-1-jn\omega_0)t}}{-1-jn\omega_0} \right|_0^1 = \frac{e^{-1-jn\omega_0}}{-1-jn\omega_0} - \frac{e^0}{-1-jn\omega_0}$$

$$D_n = \frac{e^{-1-jn\omega_0} - 1}{-1-jn\omega_0} = \frac{e^{-1-jn2\pi} - 1}{-1-jn2\pi}$$

$$D_n = \frac{-(-e^{-1} \cdot e^{-jn2\pi} + 1)}{-(1+jn2\pi)} = \frac{1 - e^{-1} e^{-j2n\pi}}{1+j2n\pi}$$

note: $e^{-j2n\pi} = \cos(2\pi n) - j\sin(2\pi n) = 1$

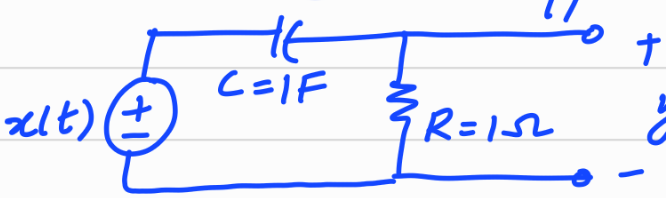
so,

$$D_n = \frac{1-e^{-1}}{1+j2n\pi}$$, And since, $x(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\omega_0 n t}$

so,

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} \left(\frac{1-e^{-1}}{1+j2n\pi} \right) e^{j2\pi n t}}$$

Pb # 03 (6.4-5): (b) $x(t)$ applied to LTIC System



$y(t) \rightarrow$ expression for output?

Transfer function of the given RC circuit is

$$H(j\omega) = \frac{1}{1+\frac{1}{j\omega}} = \frac{j\omega}{j\omega+1}$$

so,

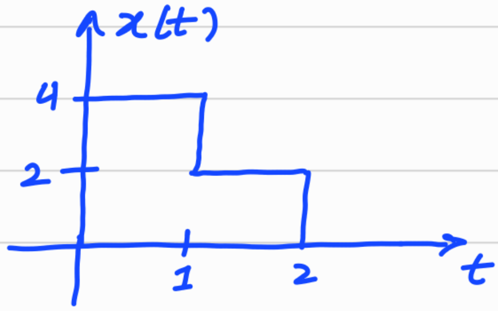
$$\Rightarrow y(t) = \sum_{n=-\infty}^{\infty} D_n H(j2\pi n) e^{j2\pi n t}$$

$$\Rightarrow y(t) = \sum_{n=-\infty}^{\infty} \left(\frac{1-e^{-1}}{1+j2\pi n}\right)\left(\frac{j2\pi n}{j2\pi n+1}\right) e^{j2\pi n t}$$

$$\Rightarrow \boxed{y(t) = \sum_{n=-\infty}^{\infty} \frac{j2\pi n(1-e^{-1})}{(1+j2\pi n)^2}\, e^{j2\pi n t}}$$

Pb#04 (7.1-6): Fourier transforms of the signals in Fig P7.1-6

Signal (a)



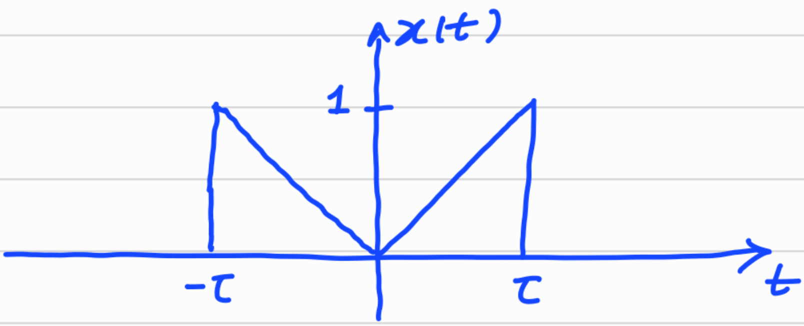
$$X(\omega) = \int_0^1 4e^{-j\omega t} + \int_1^2 2e^{-j\omega t} = \left.\frac{4e^{-j\omega t}}{-j\omega}\right|_0^1 + \left.\frac{2e^{-j\omega t}}{-j\omega}\right|_1^2$$

$$X(\omega) = \frac{4e^{-j\omega}}{-j\omega} - \frac{4e^{0}}{-j\omega} + \frac{2e^{-j2\omega}}{-j\omega} - \frac{2e^{-j\omega}}{-j\omega}$$

$$X(\omega) = \frac{-4e^{-j\omega}+4-2e^{-j2\omega}+2e^{-j\omega}}{j\omega}$$

$$\boxed{X(\omega) = \frac{4-2e^{-j\omega}-2e^{-j2\omega}}{j\omega}}$$

Signal (b)



Note: $x(t)$ is Even function so,

$$X(\omega) = \frac{2}{T}\int_0^T t\cos(\omega t)\,dt$$

→ use integration by parts

$$X(\omega) = \frac{2}{T}\left[\frac{t}{\omega}\sin(\omega t) + \frac{1}{\omega^2}\cos(\omega t)\right]\Big|_0^T$$

$$= \frac{2}{T}\left[\frac{T}{\omega}\sin(\omega T) + \frac{1}{\omega^2}\cos(\omega T) - 0 - \frac{1}{\omega^2}\right]$$

$$= \frac{2}{T}\left[\frac{\omega T\sin(\omega T) + \cos(\omega T) - 1}{\omega^2}\right]$$

$$\Rightarrow \boxed{X(\omega) = \frac{2}{T\omega^2}\left[\cos(\omega T) + \omega T\sin(\omega T) - 1\right]}$$

Pb#05: Simulink Introduction

LTI system with T.F: $H(s) = \dfrac{s+2}{s^2+2s+1}$

(a) input $x(t) = 2\sin(4t)$, $-\infty < t < \infty$
$y(t) = ?$

Compare $x(t)$ with $x(t) = A\sin(\omega t + \theta)$, gives,
$A = 2$, $\omega = 4$, $\theta = 0$

and

$$H(s) \longrightarrow H(j\omega) = H(j4)$$

$$H(j4) = \frac{j4+2}{j^2 16 + 8j + 1} = \frac{j4+2}{-16+j8+1} = \frac{j4+2}{-15+j8}$$

so,

$$|H(j4)|^2 = \frac{(2)^2+(4)^2}{(-15)^2+(8)^2} = \frac{20}{289} \Rightarrow |H(j4)| = \sqrt{\frac{20}{289}}$$

$$|H(j4)| = \frac{2\sqrt{5}}{17}$$

Also,

$$\angle H(j4) = \tan^{-1}\left(\frac{4}{2}\right) - \tan^{-1}\left(\frac{8}{-15}\right) = \tan^{-1}(2) - \tan^{-1}\left(-\frac{8}{15}\right)$$

$$\angle H(j4) = 63.4349^\circ - (-28.072^\circ + 180^\circ)$$

$63.4349^\circ - (-28.072^\circ)$

$= 91.5069$

$$\angle H(j4) = -88.4931^\circ$$

so,

$$y(t) = |H(j4)|\, 2\sin(4t + \angle H(j4))$$

so,

$$y(t) = \frac{2\sqrt{5}}{17} \cdot 2\sin(4t - 88.4931^\circ)$$

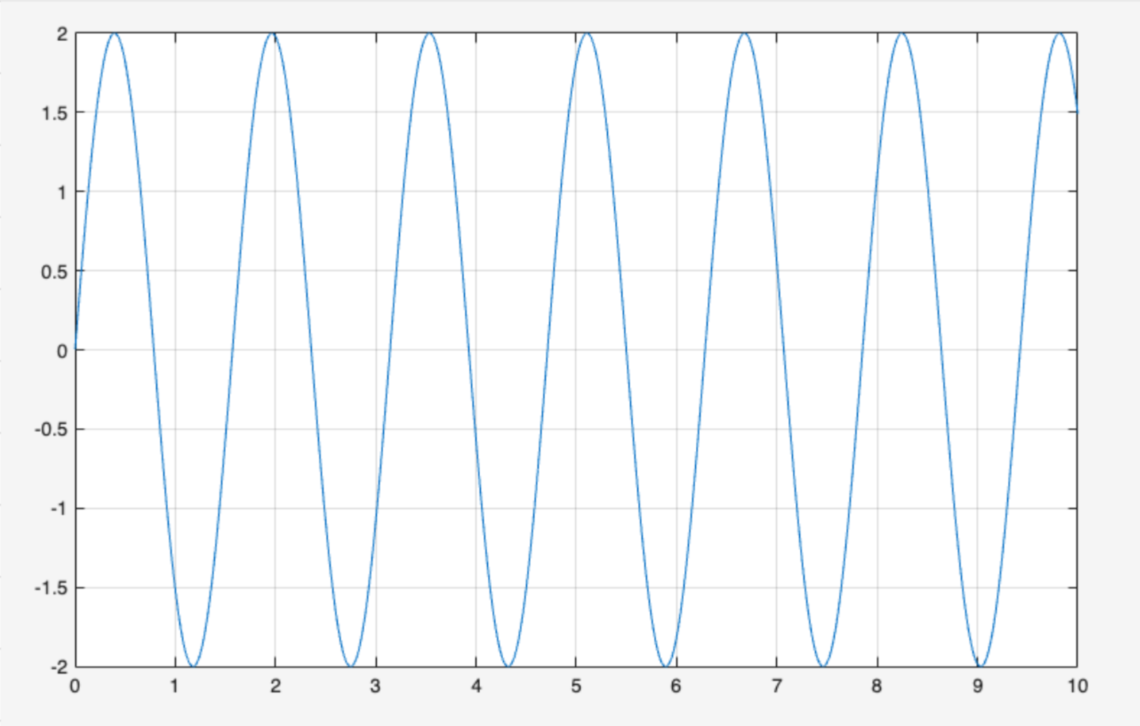
$$y(t) = \frac{4\sqrt{5}}{17}\sin(4t - 88.4931^\circ)$$

$$\boxed{y(t) = 0.5261\sin(4t - 88.4931^\circ)}$$

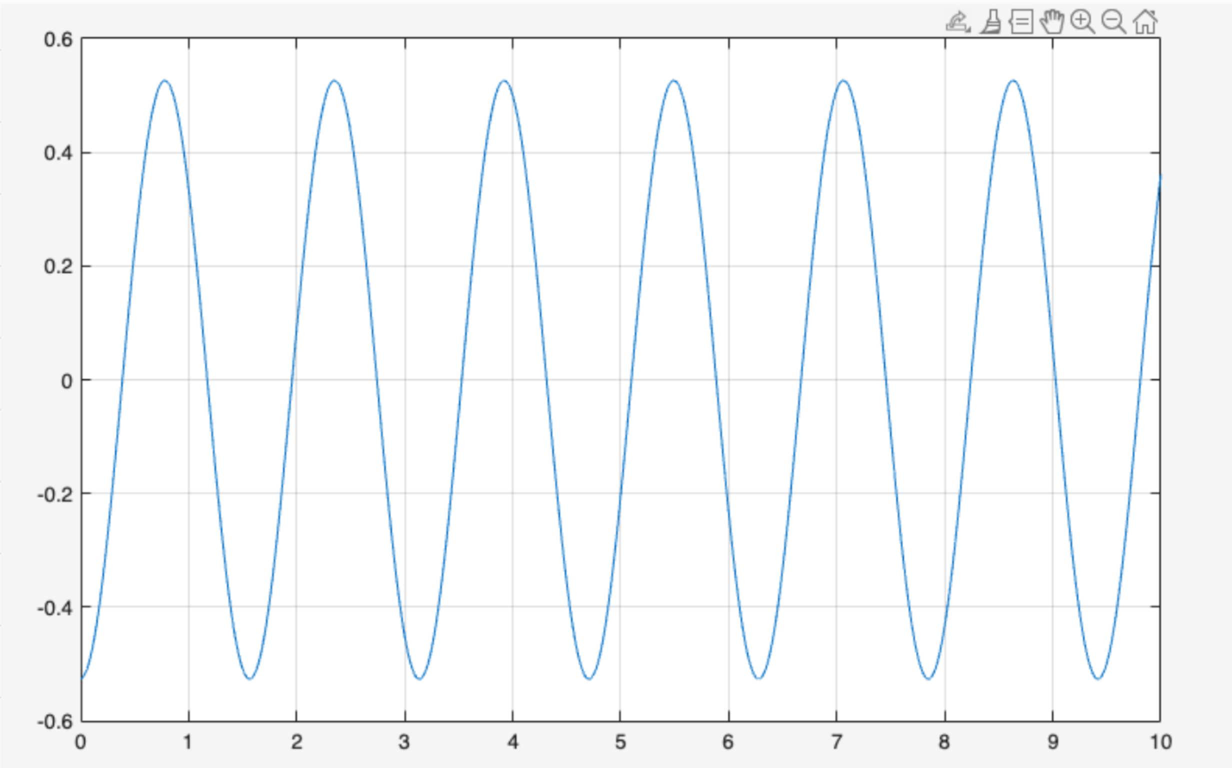
```
%% problem 5 - part a
t = 0:0.001:10;
x1 = 2*sin(4.*t);
y1_cal = 0.5261*sin(4.*t-88.4931*2*pi/360);
plot(t,x1)
grid on
figure
plot(t,y1_cal)
grid on
```

MATLAB Code.

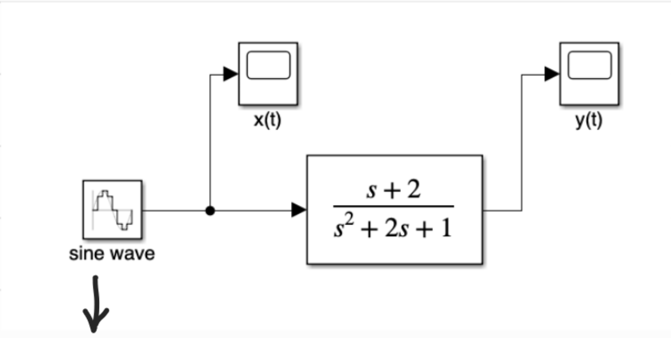
$x(t) = 2\sin(4t)$



Calculated $y(t) = 0.5261 \sin(4t - 88.4931^\circ)$



Pb#05 (b) SIMULINK MODEL:



Block Parameters: sine wave

Sine Wave

Output a sine wave:

O(t) = Amp*Sin(Freq*t+Phase) + Bias

this is how this sine wave generated block define the signal.

Sine type determines the computational technique used. The parameters in the two types are related through:

Samples per period = 2*pi / (Frequency * Sample time)

Number of offset samples = Phase * Samples per period / (2*pi)

Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

So, for $x(t) = 2\sin(4t)$

Amplitude = 2

frequency = 4

phase = Bias = 0

Parameters

Sine type: Time based

Time (t): Use simulation time

Amplitude:

2

Bias:

0

Frequency (rad/sec):

4

Phase (rad):

0

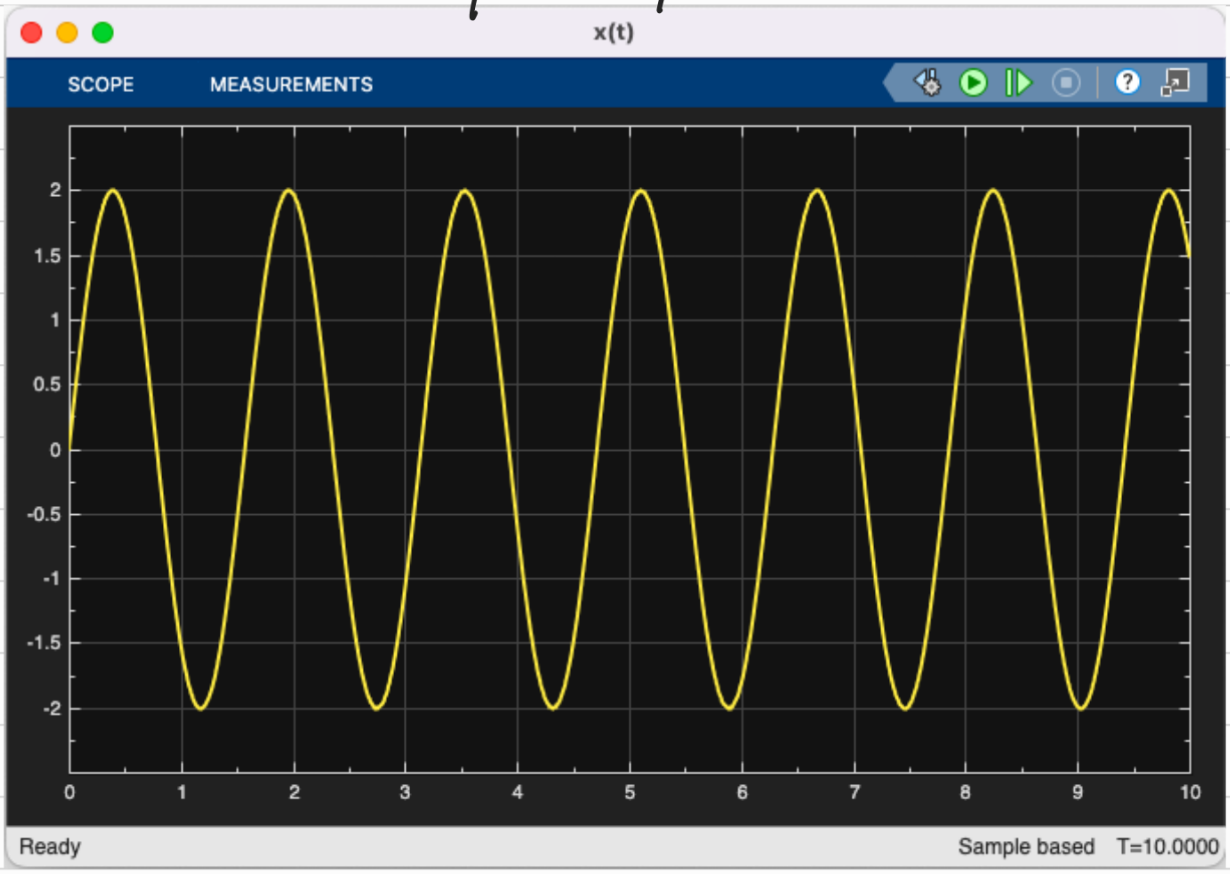
Sample time:

0.001

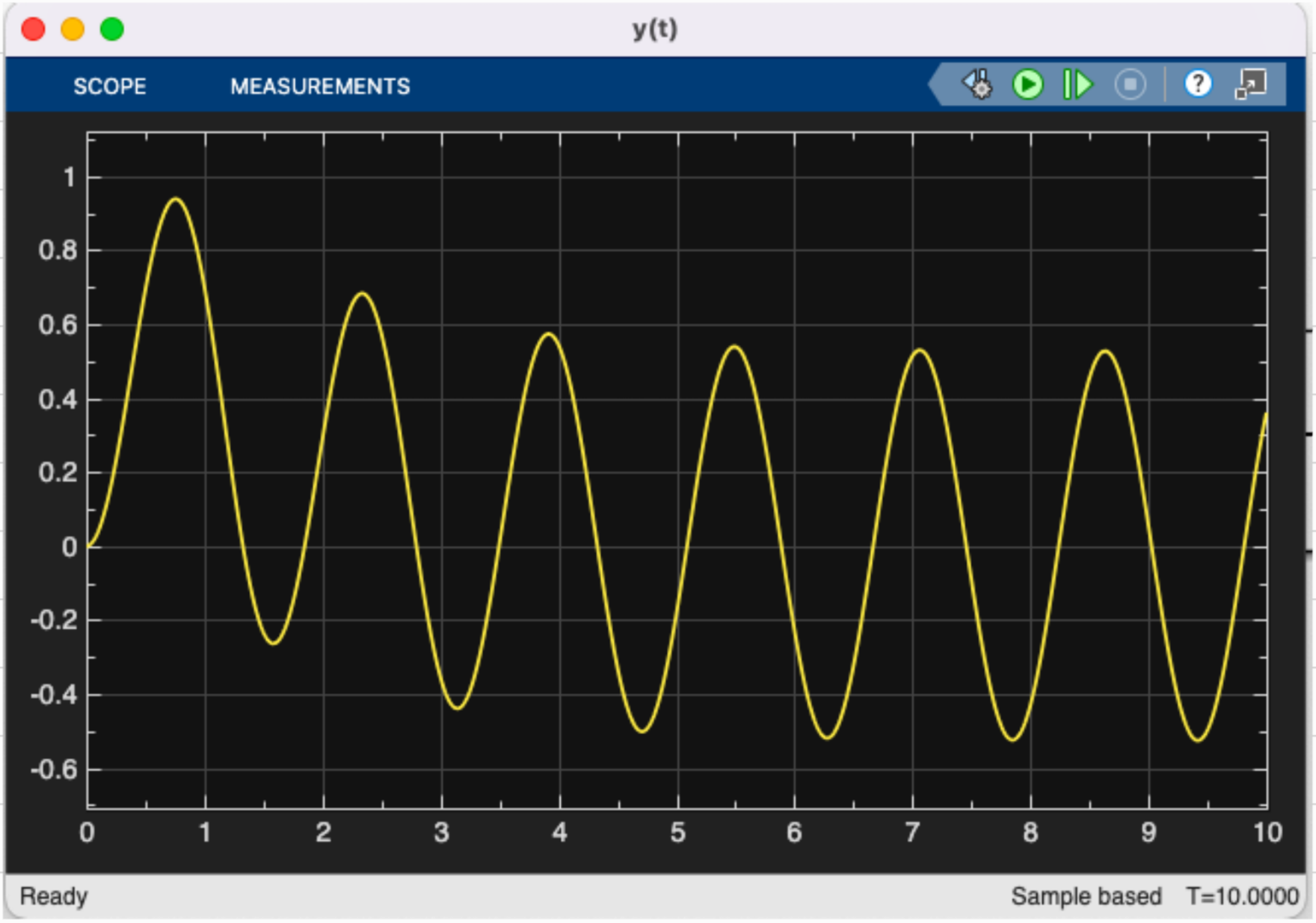
☑ Interpret vector parameters as 1-D

OK | Cancel | Help | Apply

Simulink input Scope: $x(t) = 2\sin(4t)$



Simulink output scope:



Pb#05 ⓒ: repeat ⓐ and ⓑ for $x(t) = 2\sin(100t)$

so, $A = 2$ same, but $\omega = 100$ now.

so,

$$H(s) \longrightarrow H(j\omega) = H(j100)$$

So,

$$H(j100) = \frac{j100+2}{10000j^2+200j+1} = \frac{2+j100}{-9999+200j}$$

gives,

$$|H(j100)| = \sqrt{\frac{(2)^2+(100)^2}{(-9999)^2+(200)^2}} = 0.01$$

and

$$\angle H(j100) = \tan^{-1}\left(\frac{100}{2}\right) - \tan^{-1}\left(\frac{200}{-9999}\right)$$

$$= 88.854^\circ - (-1.1458^\circ + 180^\circ)$$

$88.854^\circ - (-1.1458^\circ)$

$= 89.9998^\circ$

$$\angle H(j100) = -90.0002^\circ$$

so,

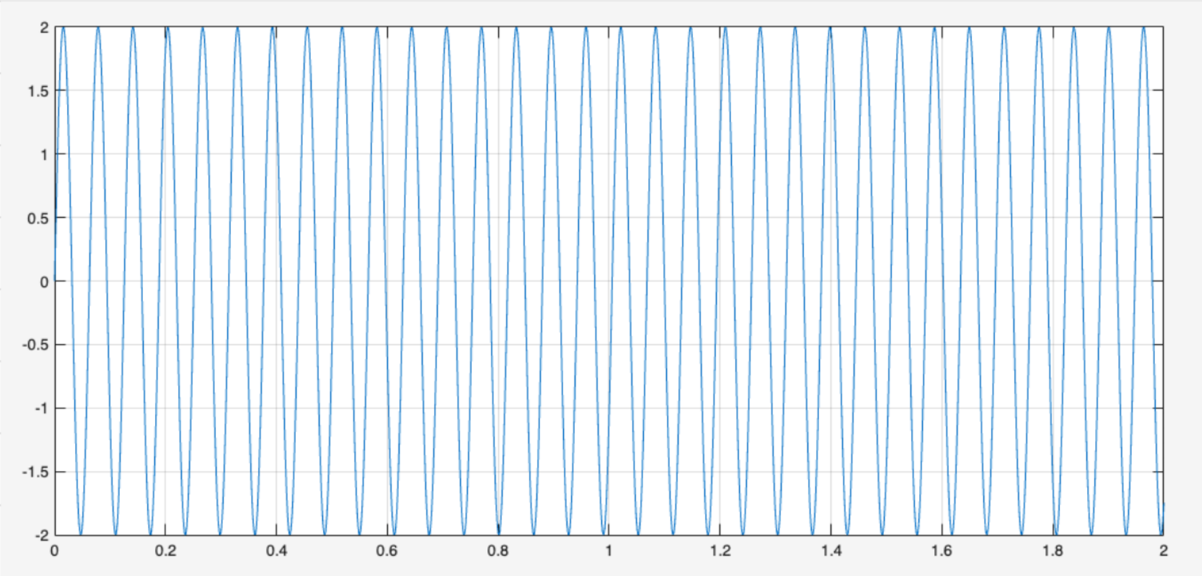
$$y(t) = |H(j100)|\, 2\sin(100t + \angle H(j100))$$

$$\boxed{y(t) = 0.02\sin(100t - 90.0002^\circ)}$$

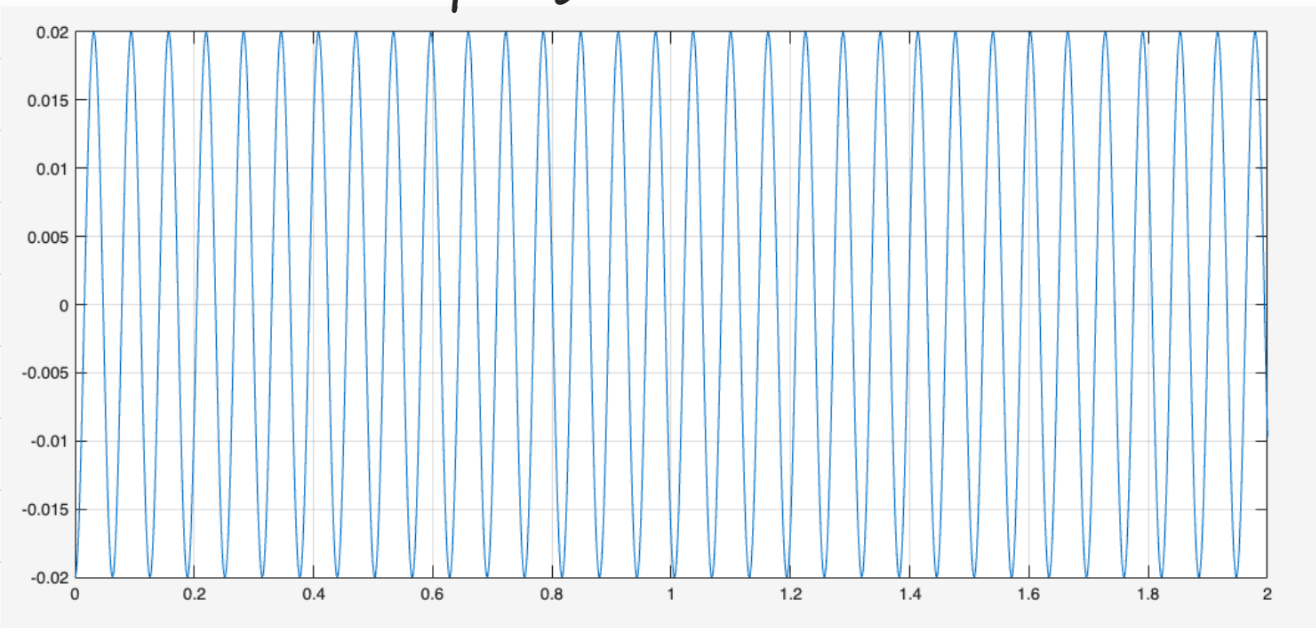
```
%% problem 5 - part c

t = 0:0.001:2;
x2 = 2*sin(100.*t);
y2_cal = 0.02*sin(100.*t-90.0002*2*pi/360);
plot(t,x2)
grid on
figure
plot(t,y2_cal)
grid on
```

→ running for less stop time

input: $x(t) = 2\sin(100t)$



Calculated output $y(t) = 0.02\sin(100t - 90.0002°)$



Simulink output Scope result: