ECE302 – HW #03 SOLUTIONS FALL 2025

Problem #01: (Lathi & Green Problem 1.5-2)

Define $x(t) = 2u(t+1) - u(t-2) - u(t-3)$

(a) Letting $x_o(t)$ designate the odd portion of $x(t)$, accurately sketch $x_o(1-2t)$.

(b) Letting $x_e(t)$ designate the even portion of $x(t)$, accurately sketch $x_e(2+t/3)$.

Solution (a): odd component: $x_o(t) = \dfrac{x(t) - x(-t)}{2}$

So, if $x(-t) = 2u(-t+1) - u(-t-2) - u(-t-3)$

then,

$$x_o(t) = \frac{2u(t+1) - u(t-2) - u(t-3) - 2u(-t+1) + u(-t-2) + u(-t-3)}{2}$$

and

$$x_o(1-2t) = \frac{2u(-2t+2) - u(-2t-1) - u(-2t-2) - 2u(2t) + u(2t-3) + u(2t-4)}{2}$$

So, $u(-2t+2) = u(-2(t-1)) = \begin{cases} 1, & t \le 1 \\ 0, & t > 1 \end{cases}$

$u(-2t-1) = u(-2(t-(-1/2))) = \begin{cases} 1, & t \le -1/2 \text{ (or } -0.5) \\ 0, & t > -1/2 \text{ (or } -0.5) \end{cases}$

$u(-2t-2) = u(-2(t-(-1))) = \begin{cases} 1, & t \le -1 \\ 0, & t > -1 \end{cases}$

$$u(2t) = u(2(t)) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$u(2t-3) = u(2(t-3/2)) = \begin{cases} 1, & t \geq 3/2 \text{ (or } 1.5) \\ 0, & t < 3/2 \text{ (or } 1.5) \end{cases}$$

$$u(2t-4) = u(2(t-2)) = \begin{cases} 1, & t \geq 2 \\ 0, & t < 2 \end{cases}$$

So for $t \leq -1$

$$x_0(1-2t) = \frac{2(1)-(1)-(1)-2(0)+(0)+(0)}{2}$$

$$x_0(1-2t) = 0$$

for $-1 < t \leq -0.5$

$$x_0(1-2t) = \frac{2(1)-(1)-(0)-2(0)+(0)+(0)}{2}$$

$$x_0(1-2t) = 0.5$$

for $-0.5 < t < 0$

$$x_0(1-2t) = \frac{2(1)-(0)-(0)-2(0)+(0)+(0)}{2}$$

$$x_0(1-2t) = 1$$

for $0 \leq t \leq 1$

$$x_0(1-2t) = \frac{2(1)-(0)-(0)-2(1)+(0)+(0)}{2}$$

$$x_0(1-2t) = 0$$

for $1 < t < 1.5$

$$x_o(1-2t) = \frac{2(0)-(0)-(0)-2(1)+(0)+(0)}{2}$$

$$x_o(1-2t) = -1$$

for $1.5 \leq t < 2$

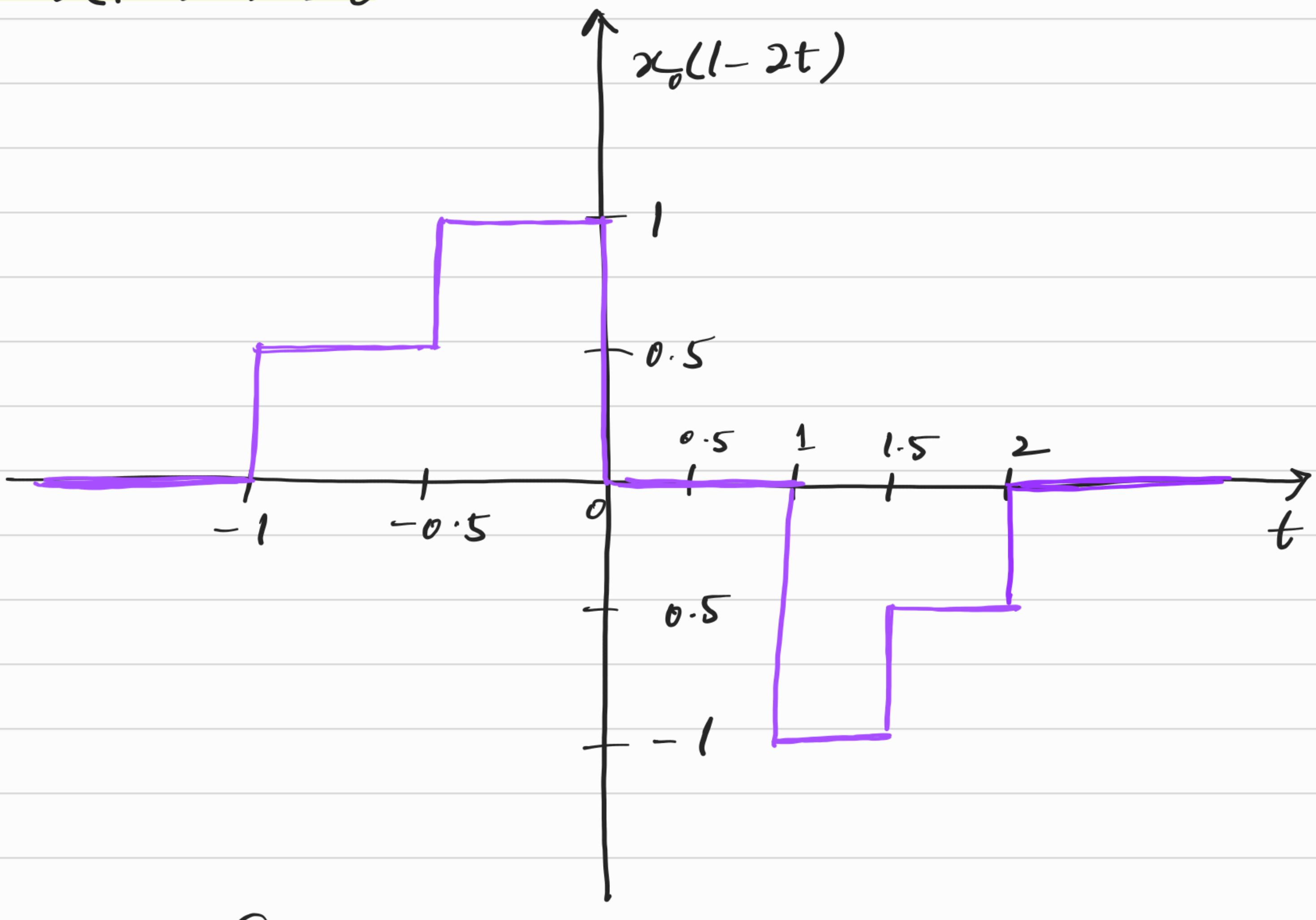
$$x_o(1-2t) = \frac{2(0)-(0)-(0)-2(1)+(1)+(0)}{2}$$

$$x_o(1-2t) = -0.5$$

and finally, for $t \geq 2$

$$x_o(1-2t) = \frac{2(0)-(0)-(0)-2(1)+(1)+(1)}{2}$$

$$x_o(1-2t) = 0$$



Solution (b): same procedure with even component

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$ to get $x_e(2+t/3)$

Note: Following is the method for Problem #01 using MATLAB.

Problem #01: (Lathi & Green Problem 1.5-2)

Define $x(t) = 2u(t+1) - u(t-2) - u(t-3)$

(a) Letting $x_o(t)$ designate the odd portion of $x(t)$, accurately sketch $x_o(1-2t)$.

(b) Letting $x_e(t)$ designate the even portion of $x(t)$, accurately sketch $x_e(2+t/3)$.

Solution (a): rewrite $x(t)$ as,

$\Rightarrow x(t) = 2u(t-(-1)) - u(t-2) - u(t-3)$

so, $x(t)$ has edges at $t=-1$ and $t=3$

Also, odd component of $x(t)$: $x_o(t) = \dfrac{x(t)-x(-t)}{2}$

$$\Rightarrow x_o(t) = \frac{2u(t+1) - u(t-2) - u(t-3) - 2u(-t+1) + u(-t-2) + u(-t-3)}{2}$$

$$= \frac{2u(t-(-1)) - u(t-2) - u(t-3) - 2u(-(t-1)) + u(-(t-(-2))) + u(-(t-(-3)))}{2}$$

$$\Rightarrow x_o(t) = u(t-(-1)) - \frac{1}{2}u(t-2) - \frac{1}{2}u(t-3) - u(-(t-1)) + \frac{1}{2}u(-(t-(-2))) + \frac{1}{2}u(-(t-(-3)))$$

so, $x_o(t)$ has edges at $t = \pm 3$.

Thus, the signal $x_o(1-2t)$ has edges at $1-2t = \pm 3$ gives,

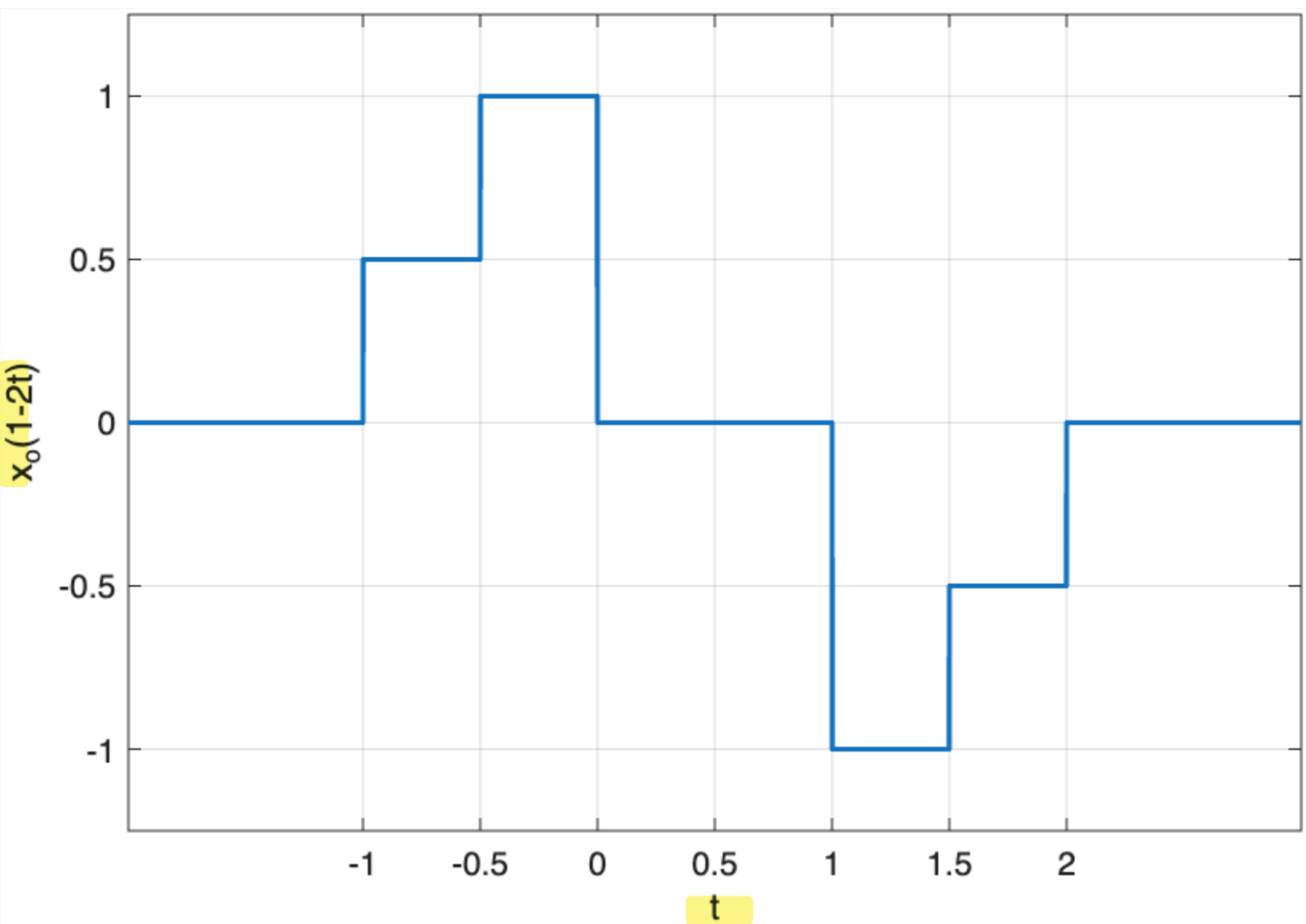
$\Rightarrow 1-2t = 3 \Rightarrow 1-3 = 2t \Rightarrow t = -2/2 \Rightarrow t = -1$

and $\Rightarrow 1-2t = -3 \Rightarrow 1+3 = 2t \Rightarrow t = 4/2 \Rightarrow t = 2$

Chose slightly wider interval of $-2 \le t \le 3$ for the plot.

MATLAB Script and plot for Pb#01(a)

```
% Define the time vector
t = -2:0.001:3;

% Define the unit step function
u = @(t) 1.0*(t >= 0);

% Define the signal x(t)
x = @(t) 2*u(t + 1) - u(t - 2) - u(t - 3);

% Compute the odd part of x(t)
xo = @(t) (x(t) - x(-t)) / 2;

% Plot the odd part of the signal with argument (1-2t)
figure;
plot(t, xo(1-2*t), 'LineWidth', 1.5);
xlabel('t');
ylabel('x_o(1-2t)');
axis([-2 3 -1.25 1.25]);
set(gca, 'XTick', -1:0.5:2, 'YTick', -1:0.5:1);
grid on;
```



Solution (b): (problem #01)

even component of $x(t)$: $x_e(t) = \dfrac{x(t) + x(-t)}{2}$

so,

$$x_e(t) = \frac{2u(t+1) - u(t-2) - u(t-3) + 2u(-t+1) - u(-t-2) - u(-t-3)}{2}$$

$$x_e(t) = u(t-(-1)) - \frac{1}{2}u(t-2) - \frac{1}{2}u(t-3) + u(-(t-1))$$
$$- \frac{1}{2}u(-(t-(-2))) - \frac{1}{2}u(-(t-(-3)))$$

Note: $x_e(t)$ also has edges at $t = \pm 3$

so, $x_e(2 + t/3)$ has edges at $2 + \frac{t}{3} = \pm 3$ given,

$\Rightarrow 2 + \frac{t}{3} = 3 \Rightarrow 6 + t = 9 \Rightarrow t = 9 - 6 \Rightarrow t = 3$

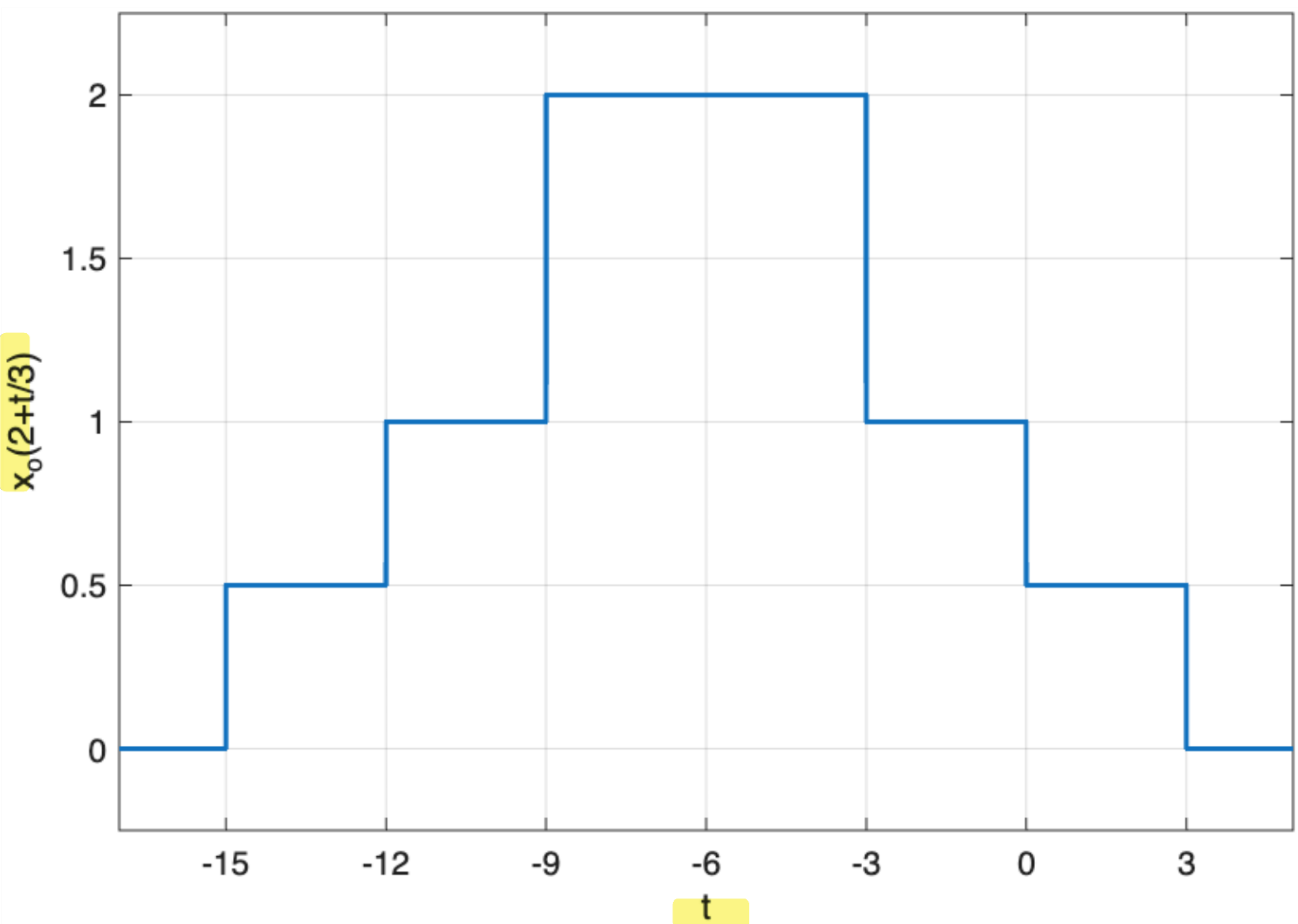
$\Rightarrow 2 + \frac{t}{3} = -3 \Rightarrow 6 + t = -9 \Rightarrow t = -9 - 6 \Rightarrow t = -15$

chose slightly wider interval of $-17 \le t \le 5$ for the plot in MATLAB.

(MATLAB Script and plot on Next page)

```
% Define the time vector
t = -17:0.001:5;

% Define the unit step function
u = @(t) 1.0*(t >= 0);

% Define the signal x(t)
x = @(t) 2*u(t + 1) - u(t - 2) - u(t - 3);

% Compute the even part of x(t)
xo = @(t) (x(t) + x(-t)) / 2;

% Plot the even part of the signal with argument (2+t/3)
figure;
plot(t, xo(2+t/3), 'LineWidth', 1.5);
xlabel('t');
ylabel('x_o(2+t/3)');
axis([-17 5 -0.25 2.25]);
set(gca, 'XTick', -15:3:3, 'YTick', 0:0.5:2);
grid on;
```

MATLAB Script and plot for Pb#01 (b)



Pbroblem #02 : (Lathi & Green problem 1.7-5)
(repeating 1.7-4)

Input $x(t)$, applied to inverting op-amp, produces output $y(t)$ according to

$$y(t+1) = \begin{cases} -2x(t) & \text{when } x(t) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

op-amp reference voltage V_{ref} and proportional delay t_r are both positive constants.

(a) Is this System BIBO Stable?

let, $|x(t)| \leq M_x < \infty$, then

$$|y(t)| \leq |-2x(t)| \leq |-2M_x| \leq 2M_x < \infty$$

Yes, Bounded input, Bounded output (BIBO) Stable.

(b) Is the system causal?

Since, output $y(t+1)$ depends of $x(t)$
and output $y(t)$ depends of $x(t-1)$
so,
No future values of input are needed to determine the output.
Yes, System is causal.

(c) Is the system Invertible?

let, $x_1(t) = -1$ and $x_2(t) = -2$, both inputs gives output $y(t+1) = 0$

Since, two different inputs yeilds the same output, so output can not be uniquely determine the input.

So,

No, the system is Not invertible.

(d) Is the system Linear?

Let $x_1(t) = 1 \Rightarrow y_1(t+1) = -2$

and $x_2(t) = -2x_1(t) = -2 \Rightarrow y_2(t+1) = 0$

Here, Homogenity (Scaling) property of linearity fails.

As for $x_2(t) = -2x_1(t)$, $y_2(t+1) \neq -2y_1(t+1) = -4$

So,

No, the system is Not Linear.

(e) Is the system Memoryless?

Since, $y(t)$ depends on $x(t-1)$ indicates system remembers past inputs.

So,

No, the system is Not Memoryless.

(f) Is the system time-invariant?

If $x(t) \longrightarrow y(t+1)$, then $x(t-T) \longrightarrow y(t+1-T)$

Since, the shift in the input causes a corresponding shift in the output. i.e $x(t) \rightarrow y(t+1) \Rightarrow x(t-1) \rightarrow y(t)$

So,

Yes, the system is time Invariant.

3. **MATLAB Introduction:**

(a) Get access to MATLAB (or some equivalent software package). See the course outline for more information on this.

(b) Read the MATLAB overview in Section B.7 of your text.

(c) Plot the function $x(t) = e^{-|t|}\cos(2\pi t)$ using MATLAB.

(d) Compute the energy in $x(t)$ using MATLAB. Check your answer by hand. Does they agree?

Remark: Always present all your simulations results in this course in a clear, concise and professional manner. This means:

1. Provide the source code.
2. Title all plots and label all axes on all plots.
3. If more than one plot is on a single graph, supply a legend.
4. Explain your results. Is it what you expect? Why? If not, why not?

An unexplained or unintelligible presentation will not be graded and will result in zero credit.

Pb #03(d) (Hand Calculation of Energy)

Since, $x(t) = e^{-|t|}\cos(2\pi t)$, $\forall t$

Note:

$$x(-t) = e^{-|-t|}\cos(-2\pi t) = e^{-|t|}\cos(2\pi t) = x(t)$$

So,

$$x(t) = 2e^{-|t|}\cos(2\pi t), \quad \forall t \geq 0$$

or Simply

$$x(t) = 2e^{-t}\cos(2\pi t), \text{ as } |t| = t \quad \forall t \geq 0$$

$$\Rightarrow E_x = \int_{-\infty}^{\infty} |x(\tau)|^2 d\tau = 2\int_0^{\infty} e^{-2\tau}\cos^2(2\pi\tau)\, d\tau$$

$$= 2\int_0^{\infty} e^{-2\tau}\left(\frac{1+\cos(4\pi\tau)}{2}\right) d\tau$$

$$\Rightarrow E_x = \left[\int_0^{\infty} e^{-2\tau}\, d\tau + \int_0^{\infty} e^{-2\tau}\cos(4\pi\tau)\, d\tau\right]$$

where, $\int_0^\infty e^{-2\tau}\, d\tau = \left.\dfrac{e^{-2\tau}}{-2}\right|_0^\infty = \dfrac{1}{2}$

and for $\int_0^\infty e^{-2\tau}\cos(4\pi\tau)\, d\tau$, use integration by parts.

$u = \cos(4\pi\tau)$, $dv = e^{-2\tau} d\tau$

$du = -4\pi \sin(4\pi\tau)\, d\tau$, $v = -\frac{1}{2}e^{-2\tau}$

$\because \int u\, dv = uv - \int v\, du$

So,

$$\int_0^\infty \cos(4\pi\tau)e^{-2\tau}\, d\tau = \left.\left(-\frac{1}{2}e^{-2\tau}\cos(4\pi\tau)\right)\right|_0^\infty$$

$$- \int_0^\infty \left(-\frac{1}{2}e^{-2\tau}\right)\left(-4\pi\sin(4\pi\tau)\right) d\tau$$

$$\int_0^\infty \cos(4\pi\tau)e^{-2\tau}\, d\tau = \frac{1}{2+8\pi^2}$$

Also note:

$$\int_0^\infty e^{-at}\cos(bt)\, dt = \frac{a}{a^2+b^2}$$

So,

$$\int_0^\infty e^{-2\tau}\cos(4\pi\tau)\, d\tau = \frac{2}{4+16\pi^2} = \frac{2}{2(2+8\pi^2)} = \frac{1}{2+8\pi^2}$$

So, $E_x = \left(\frac{1}{2} + \frac{1}{2+8\pi^2}\right) \Rightarrow \boxed{E_x = 0.5123}$

MATLAB Code for pb#03

```
%
% ECE 302 - Fall 2025
% Homework #3
% Problem 3 (c) and (d)
% MATLAB  R2025a

clc;
clear all;
close all;
clf;

% part (c)

tFinal=10;
dt=0.01;
t=[0:dt:tFinal];
x=exp(-abs(t)).*cos(2*pi*t);

figure(1);
plot(t,x,'k-');
grid on;
xlabel('t');
ylabel('x(t)');
title('plot of the function x(t)');

% part (d)

```

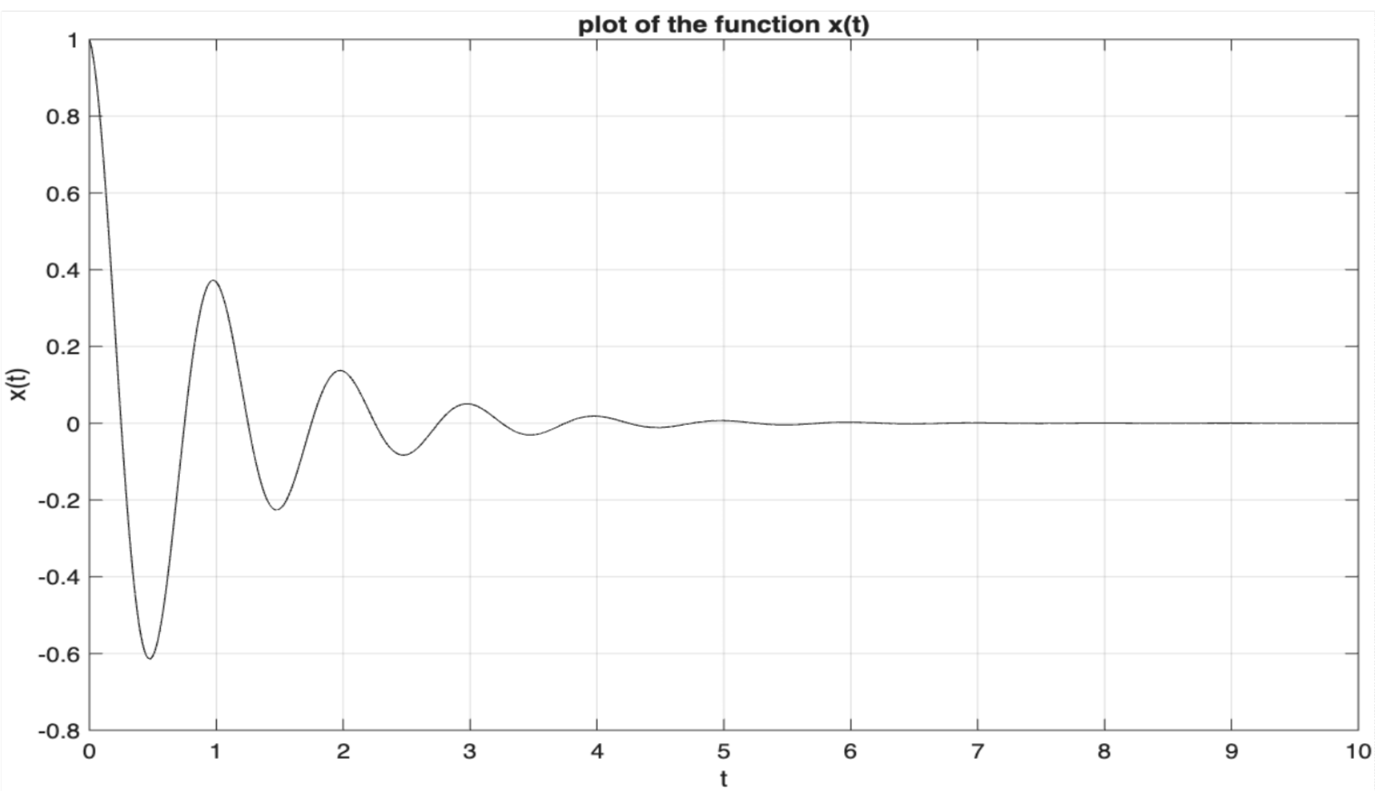
~~E=sum(x.^2).*dt~~ → E= 2* sum(x.^2).*dt

Command Window

E =

~~0.2612~~ 0.5224

Pb#03(d) Computed Energy. Note: it matched with Calculated Ex



required plot in part(c) Pb#03

Problem #04: Angles of Complex Numbers:

required angle range: $-\pi < \angle z \leq \pi$

(a) $z = -j$ Rectangular: $x + jy = 0 + j(-1) \Rightarrow x = 0, y = -1$

so,

polar: $r\angle\theta = \sqrt{x^2+y^2} \angle \tan^{-1}\left(\frac{y}{x}\right) = \sqrt{0+(-1)^2} \angle \tan^{-1}\left(\frac{-1}{0}\right) = 1\angle -90^\circ$ or $1\angle 90^\circ$

so,

$\angle z = \pm 90^\circ \times (\pi/180^\circ) \Rightarrow \angle z = \pm\pi/2$ rad. or just $\boxed{\angle z = -\pi/2}$

(b) $z = 2 + j$ Rectangle: $x + jy = 2 + j(1) \Rightarrow x = 2, y = 1$

polar: $r\angle\theta = \sqrt{(2)^2+(1)^2} \angle \tan^{-1}\left(\frac{1}{2}\right) = \sqrt{5} \angle 26.56^\circ$

so,

$\angle z = 26.56^\circ \times (\pi/180) = \boxed{0.1475\pi \text{ rad}}$ or $\boxed{0.4634 \text{ rad}}$

(c) $z = -1 - 2j$ Rectangular: $x + jy = -1 - 2j \Rightarrow x = -1, y = -2$

Polar: $r\angle\theta : \sqrt{(-1)^2+(-2)^2} \angle \tan^{-1}\left(\frac{-2}{-1}\right) = \sqrt{5} \angle 63.4349^\circ$

so,

$\angle z = 63.4349\left(\frac{\pi}{180}\right) \Rightarrow \angle z = 0.3524\pi$ rad $= 1.107$ rad

Also, $\angle z = 1.107 - \pi = \boxed{-2.03 \text{ rad}}$ As -1 shows its on $-$ve real axis

(d) $z = -1$ Rectangular: $x + jy = -1 + j(0) \Rightarrow x = -1, y = 0$

So for $x < 0, y \geq 0$, $\angle z = \tan^{-1}\left(\frac{y}{x}\right) + \pi$

so,

$\angle z = 0 + \pi \Rightarrow \boxed{\angle z = \pi}$