ECE 302 – HW #02 SOLUTIONS

Pb #01: Signal Classification:

$\boxed{a}$ $x(t) = |\sin(t)|$, $t \in \mathbb{R}$

Domain: $\mathbb{R}$ (set of all real numbers) $\rightarrow$ uncountable

for $t \in \mathbb{R}$, $\sin(t) \in [-1, 1]$

and $x(t) = |\sin(t)| \in [0, 1]$

So, co-domain : $[0, 1]$ (real numbers b/w 0 and 1) $\rightarrow$ uncountable

$x(t): \mathbb{R} \longrightarrow [0, 1]$ so, Continuous-time Analog Signal

$\boxed{b}$ $x(t) = n$, $n \leq t < n+1$, $t \in \mathbb{R}^+$ (positive real numbers), n is any non-negative integer (i.e $n \in \mathbb{Z}^{0+} = \{0, 1, 2, \dots\}$)

Domain: $t \in \mathbb{R}^+$ with $t \in [n, n+1)$

for $n = 0$, $0 \leq t < 1 \Rightarrow t \in [0, 1)$

but since $\mathbb{R}^+$ does not contain zero.

so, $t \in [n, n+1)$ for all $n \in \mathbb{Z}^+ = \{1, 2, \dots\}$

Now, set $\mathbb{Z}^+$ (+ve integers) is infinitely countable

but because t is any positive real number in the interval $[n, n+1)$ is subset of $\mathbb{R}^+ \longrightarrow$ uncountable

So, Domain $t \in [n, n+1) \subseteq \mathbb{R}^+$, $n \in \mathbb{Z}^+ \longrightarrow$ uncountable

and

Co-domain: $n \in \mathbb{Z}^{0+} \rightarrow$ infinitely countable

so,

$x(t): [0, n+1) \subseteq \mathbb{R}^+ \longrightarrow \mathbb{Z}^{0+}$ so, Continuous-time, digital signal

[c] $x(n) = 2^n$ for each integer n.

Domain: $n \in \mathbb{Z}$ (set of integers) $\rightarrow$ countable

codomain: $\{ \cdots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \cdots \}$

Set of positive rational $\mathbb{Q}^+$ and +ve natural numbers $\mathbb{N}^+$.
both $\mathbb{Q}^+$ and $\mathbb{N}^+$ are countable

So, codomain: is countable

and so, $x(n) = 2^n$ is Discrete-time, Digital Signal

Pb#02: Signal Models:

[a] $x(t) = u(2t) + 1$

unit step function

$$u(t) = \begin{cases} 0 ; & t < 0 \\ 1 ; & t \geq 0 \end{cases}$$

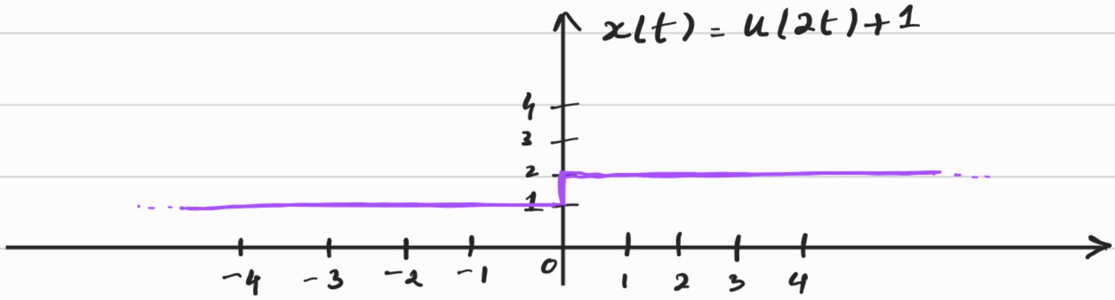
$$u(2t) = \begin{cases} 0 ; & 2t < 0 \\ 1 ; & 2t \geq 0 \end{cases}$$

$$u(2t) = \begin{cases} 0 ; & t < 0 \\ 1 ; & t \geq 0 \end{cases}$$

Suggest $u(2t) = u(t)$

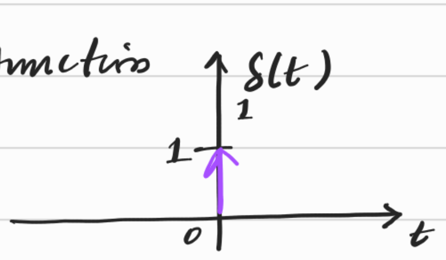
Now,

$$u(2t) + 1 = \begin{cases} 1 ; & t < 0 \\ 2 ; & t \geq 0 \end{cases}$$

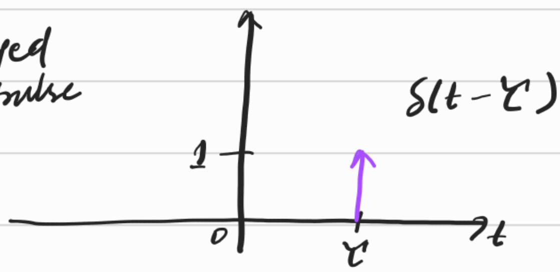


[b] $x(t) = 2\delta(t-2) - 3\delta(t-1)$

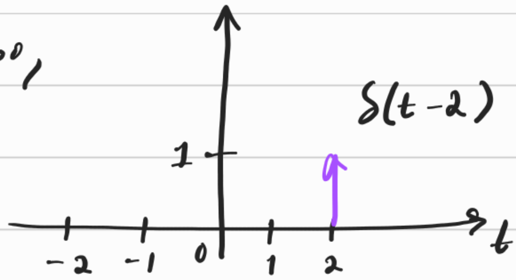
impulse function



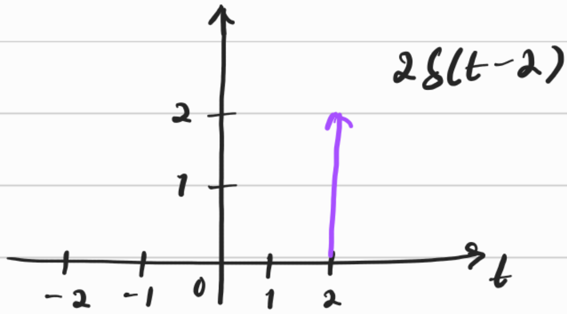
Delayed impulse



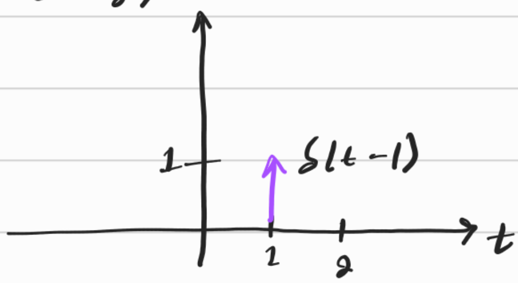
So,



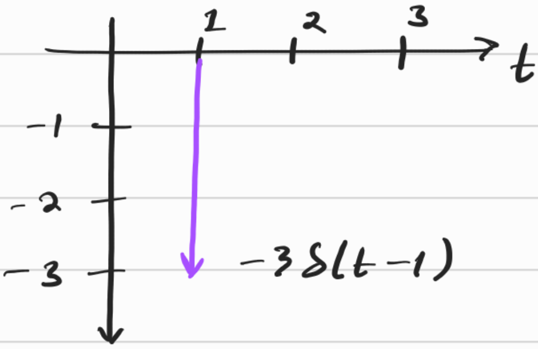
$\Rightarrow$



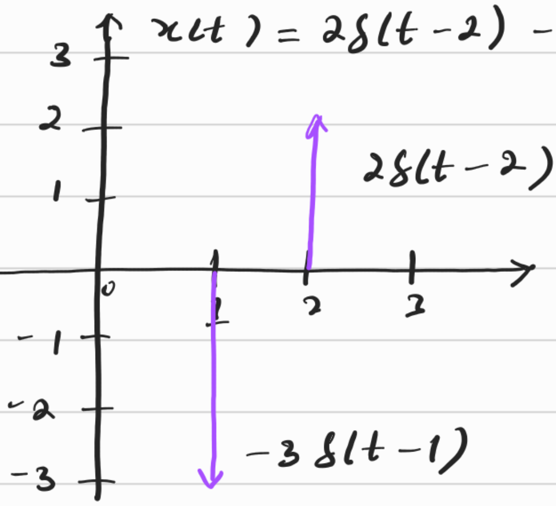
Similarly,



$\Rightarrow$



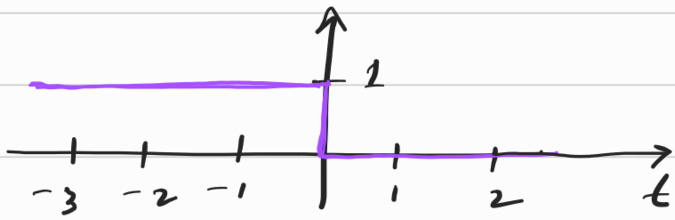
So, $x(t) = 2\delta(t-2) - 3\delta(t-1)$



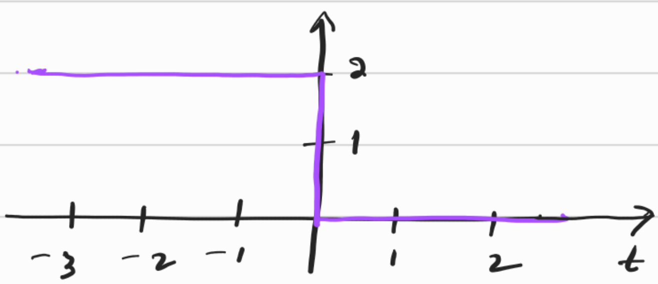
$\boxed{1c}$ $x(t) = \delta(t-1) + 2u(-t)$

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \Rightarrow u(-t) = \begin{cases} 0, & -t < 0 \\ 1, & -t \geq 0 \end{cases}$$

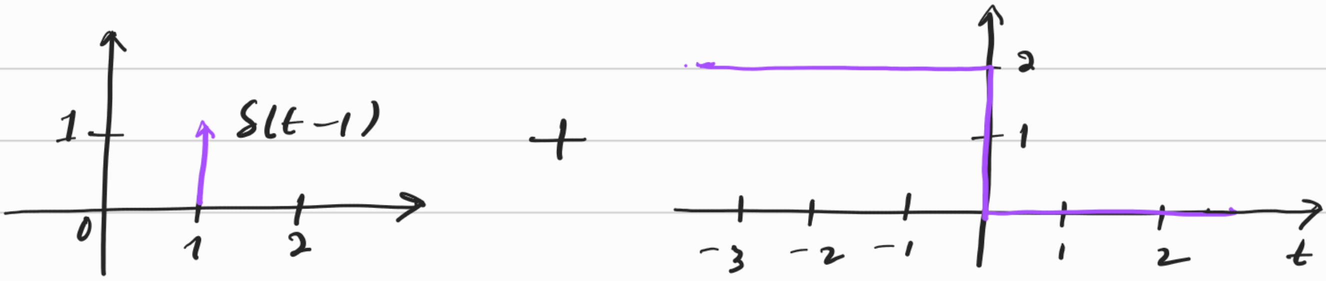
$$\Rightarrow u(-t) = \begin{cases} 0, & t > 0 \\ 1, & t \leq 0 \end{cases} \Rightarrow$$



$$\Rightarrow 2u(-t) = \begin{cases} 0, & t > 0 \\ 2, & t \leq 0 \end{cases} \Rightarrow$$

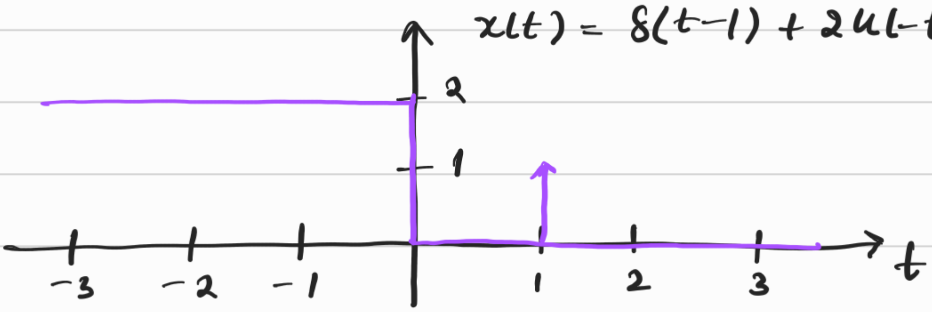


So,



$x(t) = \delta(t-1) + 2u(-t)$

gives,



Pb#03: Properties of Impulse Functions:

[a] $(at^2 + bt + c)\,\delta(t+1)$, where $a, b, c \in \mathbb{R}$

property of $\delta(t)$: $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

we have $\delta(t+1) = \delta(t-(-1)) \Rightarrow t_0 = -1$

So, if $x(t) = at^2 + bt + c \Rightarrow x(-1) = a - b + c$

and so, simplified expression is

$\Rightarrow \boxed{(a-b+c)\,\delta(t+1)}$, for $a, b, c \in \mathbb{R}$

[b] $e^{-j\pi t}\left[\delta(t-2) - \delta(t+2)\right]$

→ Expand

$\Rightarrow \quad e^{-j\pi t}\delta(t-2) - e^{-j\pi t}\delta(t+2)$

So, we have,

$x_1(t)\delta(t-t_1) - x_2(t)\delta(t-t_2)$, with $x_1(t) = x_2(t) = e^{-j\pi t}$

from

$\delta(t-t_1) = \delta(t-2) \Rightarrow t_1 = 2$, So, $x_1(t_1) = e^{-j2\pi}$

or $x_1(t_1) = \cos(2\pi) - j\sin(2\pi) \Rightarrow x_1(t_1) = 1$

And from

$\delta(t-t_2) = \delta(t+2) \Rightarrow t_2 = -2$, So, $x_2(t_2) = e^{j2\pi}$

and so, using Euler's identity again, we get,

$$x_2(t_2) = \cos(2\pi) + j\sin(2\pi) \Rightarrow x_2(t_2) = 1$$

so, if

$$x_1(t)\delta(t-t_1) - x_2(t)\delta(t-t_2) \Rightarrow x_1(t_1)\delta(t-t_1) - x_2(t_2)\delta(t-t_2)$$

Then,

$$e^{-j\pi t}\delta(t-2) - e^{-j\pi t}\delta(t+2) \Rightarrow \boxed{\delta(t-2) - \delta(t+2)}$$

$\boxed{c}$ $\sum_{k=1}^{\infty} \frac{1}{t}\delta(t-k)$

so, $\delta(t-t_0) = \delta(t-k) \Rightarrow t_0 = k$

and if $x(t) = \frac{1}{t} \Rightarrow x(t_0) = \frac{1}{k}$

and so, $x(t)\delta(t-t_0) \Rightarrow x(t_0)\delta(t-t_0)$

we get, $\sum_{k=1}^{\infty} \frac{1}{t}\delta(t-k) \Rightarrow \boxed{\sum_{k=1}^{\infty} \frac{1}{k}\delta(t-k)}$

Pb#04: Impulse functions and Integrals:

$\boxed{a}$ $\int_{-\infty}^{\infty} \cosh(2\pi\tau)\delta(\tau-2)\,d\tau$

property of $\delta(t)$: $\int_{-\infty}^{\infty} x(\tau)\delta(\tau-t_0)\,d\tau = x(t_0)$

from $\delta(\tau-2) = \delta(\tau-t_0) \Rightarrow t_0 = 2$ and if $x(t) = \cosh(2\pi\tau)$

Then,

$$\Rightarrow \int_{-\infty}^{\infty} \cosh(2\pi\tau)\delta(\tau-2)\,d\tau = \cosh(2\pi(2))$$

$$= \boxed{\cosh(4\pi)} = 143375.65657\ldots$$

$\boxed{b}$ $\int_{-\infty}^{t} \cosh(2\pi\tau)\delta(\tau-2)\,d\tau$

from $\delta(\tau-2)$, we have same $t_0 = 2$

property: $\int_{-\infty}^{t} x(\tau)\delta(\tau - t_0)\,d\tau = \begin{cases} x(t_0) & ; \; t \geq t_0 \\ 0 & ; \; t < t_0 \end{cases}$

so

$$\Rightarrow \int_{-\infty}^{t} \cosh(2\pi\tau)\delta(\tau-2)\,d\tau = \begin{cases} \cosh(4\pi) & ; \; t \geq 2 \\ 0 & ; \; t < 2 \end{cases}$$

[c] $\int_{t}^{\infty} \cosh(2\pi\tau)\delta(\tau-2)\,d\tau$, again $t_0 = 2$

property: $\int_{t}^{\infty} x(\tau)\delta(\tau - t_0)\,d\tau = \begin{cases} x(t_0) & ; \; t \leq t_0 \\ 0 & ; \; t > t_0 \end{cases}$

so,

$$\Rightarrow \int_{t}^{\infty} \cosh(2\pi\tau)\delta(\tau-2)\,d\tau = \begin{cases} \cosh(4\pi) & ; \; t \leq 2 \\ 0 & ; \; t > 2 \end{cases}$$

Pb#05: Rectangular/Polar Forms of Complex Numbers:

[a] $2 - j3$

(rectangular form)

compare with: $x + jy$

to get, $x = 2$, $y = -3$

and polar form: $r \angle \theta$

where,

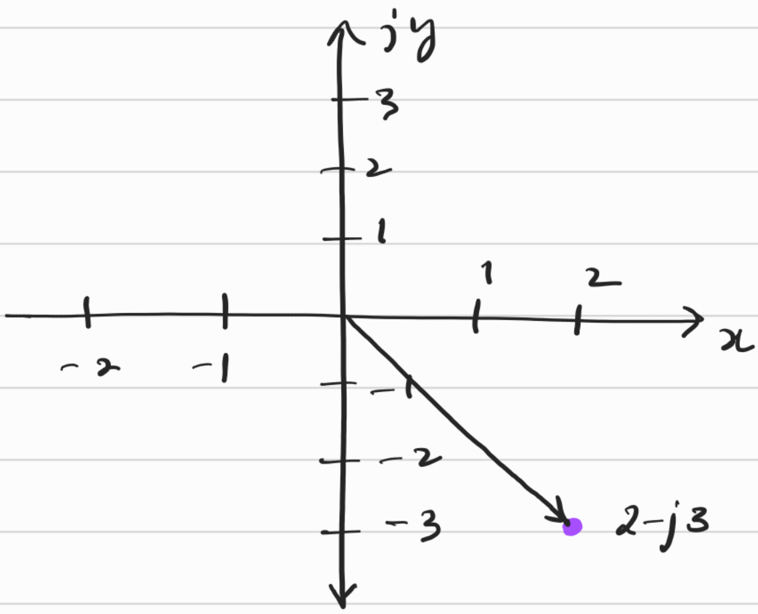
$r = \sqrt{x^2+y^2} = \sqrt{13} = 3.605$

and $\theta = \tan^{-1}\left(\frac{y}{x}\right) = -56.309^\circ$

so,

$2 - j3 \Rightarrow \boxed{3.605 \angle -56.309^\circ}$

required $\boxed{\text{polar form}}$



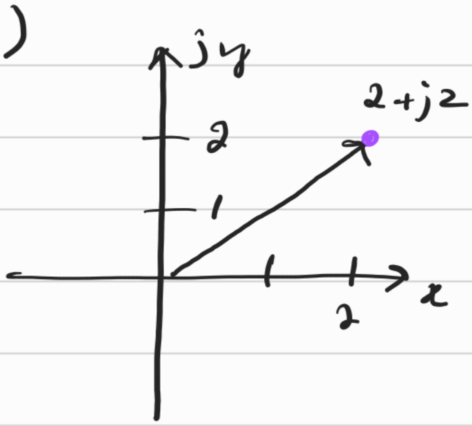
1b $2+j2$ (rectangular form) $(x+jy)$

give $x=2$, $y=2$

so, $r=\sqrt{x^2+y^2}=2\sqrt{2}=2.828$

and $\theta=\tan^{-1}\left(\frac{y}{x}\right)=45^\circ$

So, $\boxed{\text{polar form}}$: $r\angle\theta=\boxed{2.828\angle 45^\circ}$



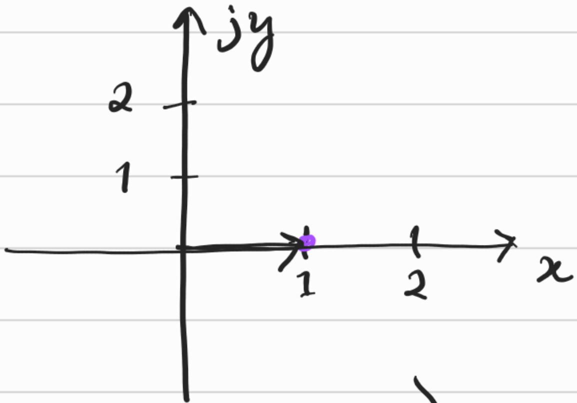
1c $e^{j6\pi}$ (polar form)

Since,

$re^{j\theta}=e^{j6\pi} \Rightarrow r=1,\ \theta=6\pi$

and

$x=r\cos\theta$	$y=r\sin\theta$
so, $x=\cos(6\pi)$	$y=\sin(6\pi)$
$x=1$	$y=0$

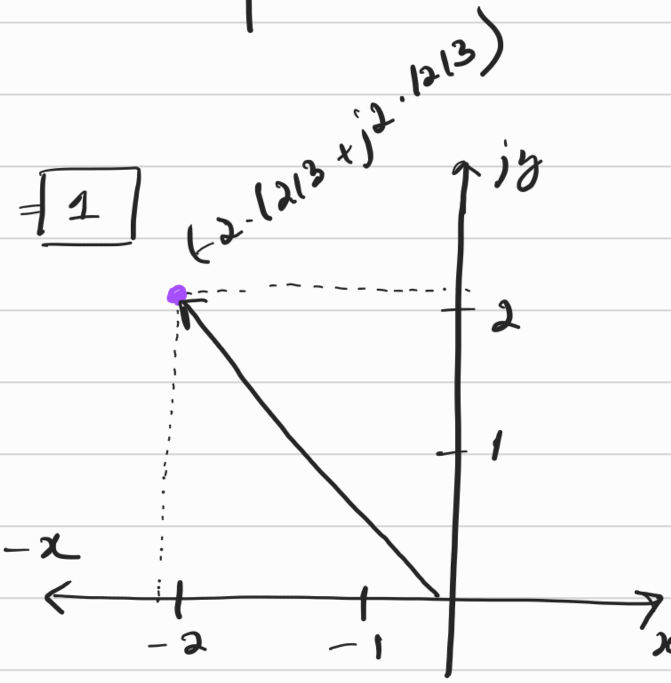


so,

$\boxed{\text{rectangular form}}$: $x+jy=1+j0=\boxed{1}$

1d $3e^{-j5\pi/4}$ (polar form)

where, $r=3$, $\theta=\frac{-5\pi}{4}$



so,

$x=r\cos\theta$	$y=r\sin\theta$
$x=3\cos\left(\frac{-5\pi}{4}\right)$	$y=3\sin\left(\frac{-5\pi}{4}\right)$
$x=3\left(\frac{-\sqrt{2}}{2}\right)$	$y=3\left(\frac{\sqrt{2}}{2}\right)$
$x=-2.1213$	$y=2.1213$

So, $\boxed{\text{rectangular form}}$: $x+jy=\boxed{-2.1213+j2.1213}$