Kalman Filter for Interference Mitigation and Channel Equalization in Aeronautical Telemetry

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Abstract—In this paper we describe a new method that is applicable to mitigating both multipath interference and adjacent channel interference (ACI) in aeronautical telemetry applications using ARTM Tier-1 waveforms. The proposed method uses a linear equalizer that is derived using Kalman filtering theory, which has been used for channel equalization for high-speed communication systems. We illustrate the proposed method with numerical examples obtained from simulations that show the bit error rate performance (BER) for different modulation schemes.

I. INTRODUCTION

Data rates required in modern aeronautical telemetry applications have increased, thus generating more demand for bandwidth for telemetry signals. As a consequence new modulation schemes with improved spectral efficiency have been proposed for use in aeronautical telemetry. Among these we note the Fekher-patented QPSK modulation (FQPSK) [1] and the Shaped-Offset QPSK modulation (SOQPSK) [2] also known as Advanced Range Telemetry (ARTM) Tier-1 Waveforms, which consist of variations of QPSK modulation schemes that encode 2 bits/symbol.

An additional requirement for efficient spectral usage is that the carriers assigned to distinct telemetry signals be spaced as closely as possible. As the carrier spacing is decreased, the spectra of corresponding signals will start overlapping creating ACI which is a limiting factor in carrier spacing assignments. We note that the carrier spacing along with the relative powers of signals in adjacent channels determine the amount of ACI affecting a desired telemetry signal.

To mitigate the effects of ACI minimum carrier spacings have been recommended for telemetry signals [3]. Recently, the use of interference cancellation techniques has also been investigated as an alternative to enable multiple telemetry signals to be packed more closely together in order to improve the overall efficiency with which the frequency band allocated for telemetry applications is used [4].

Another consequence of the increasing data rates in aeronautical telemetry applications is the fact that the multipath interference affecting propagation of telemetry signals has become more frequency selective as shown by the recently developed channel models for aeronautical telemetry [5]. To mitigate the effects of multipath interference two equalization techniques are investigated in [6]: the constant modulus algorithm (CMA), and the decision-feedback minimum mean square error (DF-MMSE) algorithm.

In our paper we present a new method that is applicable to mitigating both multipath interference and ACI using a linear equalizer based on Kalman filtering theory [7]. We note that Kalman-based equalizers have long been used in communication systems [8], [9]. The paper is organized as follows: in Section II we describe the ARTM Tier-1 Waveforms used for digital modulation in aeronautical telemetry applications. In Section III we describe the proposed equalizer for ARTM Tier-1 waveforms, followed by presentation of numerical results obtained from simulations in Section IV and final remarks and conclusions in Section V.

II. ARTM TIER-1 WAVEFORMS USED IN AERONAUTICAL TELEMETRY APPLICATIONS

A. FQPSK Modulation

FQPSK modulation [1] is a variant of offset QPSK modulation in which the inphase and quadrature components of the modulated waveforms are cross correlated to produce a signal with quasi-constant envelope. The complex baseband FQPSK waveform is expressed in terms of a set of a set of M = 16 baseband pulses $S_m(t)$, $m = 0, \ldots, M - 1$, and is represented as

$$f(t) = \sqrt{E_b} \sum_n \{ S_{m(n)}(t - nT_s) + jS_{q(n)}(t - (n - 0.5)T_s) \}$$

where $E_b$ represents the average bit energy and $T_s$ is the symbol duration. During the symbol interval $nT_s \leq t \leq (n + 1)T_s$ the waveform $S_{m(n)}(t - nT_s)$ is used to perform amplitude modulation of the inphase component of the carrier, while during the interval $(n + 0.5)T_s \leq t \leq (n + 1.5)T_s$ the waveform $S_{q(n)}[t - (n - 0.5)T_s]$ is used to perform amplitude modulation of the quadrature component of the carrier, and indices $i(n), q(n) \in \{0, \ldots, M - 1\}$ are determined by the input data streams as described in [10].

The optimal detector for FQPSK modulation is a sequence detector that uses a trellis which accounts for all possible combinations of waveforms determined by the memory of the waveform mapper [10]. However, in practice a symbol-by-symbol detector as shown in Figure 1 is used. This type of detector can be used also for SOQPSK modulated signals, and its performance is very close to that of the trellis detector.
B. SOQPSK Modulation

SOQPSK modulation [2] is a ternary continuous phase modulation (CPM) scheme with modulation index equal to 1/2, for which the baseband SOQPSK waveform is represented as

$$s(t) = e^{j\phi(t)}$$

with phase $\phi(t)$ expressed as

$$\phi(t) = \pi \sum_k \alpha(k) g(t - kT_b)$$

where $\alpha(k) \in \{-1, 0, +1\}$ is the $k^{th}$ ternary symbol, $T_b$ is the bit duration, and $g(t)$ is a phase pulse that is the time integral of a frequency pulse $p(t)$ with area equal to 1/2. In our paper we considered the SOQPSK-TG waveforms as defined in [11], [12], and for SOQPSK waveforms was discussed in [13].

III. THE PROPOSED EQUALIZER FOR ARTM SIGNALS

Let $r(t)$ be the received baseband signal corresponding to a desired transmitted ARTM signal that employs FQPSK or SOQPSK modulation as described in the previous section, and which is affected by ACI and/or multipath. To mitigate the interference effects on the received signal $r(t)$ we propose the use of a linear $N$-tap delay line filter inspired from the Kalman-filter based channel equalizer proposed by Godard in 1974 [9], that operates on the sampled received baseband ARTM signal $r(n)$. We denote the filter tap weights vector by

$$c = [c_0 \ldots c_N]^\top$$

with the corresponding tap output vector denoted by

$$r_n = [r(n) r(n-1) \ldots r(n-N)]^\top$$

Then, the equalized signal is expressed as

$$y_n = r_n^\top c$$

The values of the equalizer taps are obtained during a training stage in which a set of samples of the desired signal $\{a_n\}$ known at the receiver is transmitted. The optimal tap values must minimize the expected mean squared distortion $e_n$ between the training sample and the output of the equalizer, that is

$$E = E[|a_n - y_n|^2] = E[|e_n|^2]$$

According to [9], the optimum tap weight vector is given by

$$c^* = B^{-1}b$$

with $B = E[r_n r_n^\top]$ and $b = E[a_n r_n]$, and when $c$ is chosen to be equal to $c^*$ the mean-squared distortion is minimized and equal to $E^*$. We note that, even when no noise is present, $E^* \neq 0$ due to the fact that the equalizer has a finite impulse response, while an infinite impulse response filter is necessary to equalize a finite impulse response channel. We denote by $e_n^*$ the distortion when the optimal tap weights are user, and we write the training symbols as

$$a_n = r_n^\top c^* + e_n^*$$

with $E[e_n^*^2] = E^*$. With random initialization of the tap weights vector, the dynamic evolution of the optimal tap weights vector $c$ during the training stage is described by the following state-space model

$$c_n = c_{n-1} + a_n^\top c_n + e_n$$

in which the state transition matrix is the identity matrix $I$, and the noisy measurement equation expresses the value of training sample $a_n$ at time instant $n$ in terms of the actual output of the equalizer $r_n^\top c_n$, at time instant $n$ and the expected distortion $e_n$. The fact that the state transition matrix is equal to the identity matrix implies that at steady state the value of the optimal tap weights vector does not change, and is essentially constant [9].
The optimal tap weights are obtained by applying the Kalman filtering algorithm as in [9], which is different from [8] in which the Kalman filter is actually the equalizer. We recall that, for a general linear system described by the state-space equations

\[ x_n = A_{n,n-1}x_{n-1} + w_n \]
\[ y_n = C_nx_n + v_n \]  

(14)

where

- \( x_n \) system state vector
- \( A_{n,n-1} \) is the state transition matrix
- \( w_n \) state noise vector with covariance matrix \( Q_n \)
- \( C_n \) is the measurement matrix
- \( v_n \) measurement noise vector with covariance matrix \( R_n \)

the discrete-time Kalman filtering equations [7] for estimation of state vector \( \mathbf{x} \) are

1) predicted state estimate (a priori estimate)

\[ \hat{x}_n^- = A_{n,n-1}\hat{x}^+_n-1 \]  

(15)

2) predicted measurement equation

\[ \hat{y}_n = C_n\hat{x}_n^- \]  

(16)

3) error covariance extrapolation

\[ P_n^- = A_{n,n-1}P^+_n-1 A_{n,n-1}^T + Q_{n,n-1} \]  

(17)

4) Kalman gain matrix equation

\[ K_n = P_n^-C_n^T(C_nP_n^-C_n^T + R_n)^{-1} \]  

(18)

5) error covariance update equation

\[ P^+_n = (I - K_nC_n)^{-1}P_n^- \]  

(19)

6) state estimate update (a posteriori estimate)

\[ \hat{x}_n^+ = \hat{x}_n^- + K_n(y_n - \hat{y}_n) \]  

(20)

For the particular case of the linear system in equation (13) which describes the evolution of the optimal tap weights during the training stage, there is no state noise (\( Q_n = 0 \)), and the (scalar) measurement noise has variance \( R_n = E^* \). The discrete-time Kalman filtering equations (15) – (20) imply that the optimal estimate of the tap weights vector is

\[ \hat{a}_n = r_n^T\hat{e}_n-1 \]
\[ K_n = P_n^0r_n(r_n^TP_n^0r_n + E^*)^{-1} \]
\[ P_n^0 = (I - K_nr_n^T)r_n^T \]
\[ \hat{e}_n = \hat{e}_n-1 + K_n(a_n - \hat{a}_n) \]  

(21)

We note that in practice the value of \( E^* \) cannot be known a priori, and in order to compute the Kalman gain \( K_n \) an estimated value \( \hat{E}^* \) is used. According to [9] this has no influence on the successive estimates of the tap weights, and is usually taken between \( 10^{-3} \) and \( 10^{-2} \).

After the training stage is completed and steady state is reached, we use the steady state value \( \hat{e} \) to equalize the received ARTM signal and estimate transmitted symbols using the equalized signal.

**Fig. 2.** BER performance for FQPSK signals with multipath channel.

**Fig. 3.** BER performance for SOQPSK signals with multipath channel.

**IV. Simulation Results**

We have performed simulations to evaluate the improvements in BER when the equalizer filter described in the previous section is used for mitigating ACI and multipath interference in aeronautical telemetry systems using FQPSK and SOQPSK modulation. The data rate for both modulation schemes was taken 10 Mbps, and the received signal was sampled at 10 samples/symbol.

For the case of multipath we considered a two-ray propagation channel model which is standard for aeronautical telemetry systems [5], and which was used also in the equalization studies in [6]. We have simulated the system for several values of the ground reflected path gain, and simulation results are presented in Figure 2 (for FQPSK modulation), respectively Figure 3 (for SOQPSK modulation). The equalizers used 1000 samples for training (corresponding to 100 transmitted symbols), and their lengths for \( \Gamma = 0.3, 0.5, 0.7 \) were \( N = 5, 7, 10 \) taps respectively for FQPSK signals, and \( N = 3, 5, 7 \).
no multipath are presented in Figure 4, and with the same two ACI signals but with multipath with $\Gamma = 0.3$ for all channels are presented in Figure 5. In both cases a $N = 5$—tap equalizer filter with 1000 samples for training (corresponding to 100 transmitted symbols) yielded the best performance improvement, and we note that in this case the Kalman-based equalizer filter provides performance very close to that when no ACI is present. We also note that all ACI signals were assumed synchronized with the desired signal, and that there is about 1 dB loss in performance when synchronization is no longer assumed. In the case of SOQPSK modulation however, the proposed Kalman-based equalizer didn’t yield significant improvements.

V. CONCLUSIONS

In this paper we presented a new method for mitigating ACI and multipath interference in aeronautical telemetry applications using ARTM Tier-1 waveforms. The proposed method uses a linear filter which has been used for channel equalization for high-speed communication systems and which is obtained by applying Kalman filtering techniques, and improves BER performance in the presence of multipath. In addition, in the case of FQPSK modulation it can also be used for ACI mitigation.

REFERENCES