Game-Theoretic Approach to Joint Transmitter Adaptation and Power Control in Wireless Systems

Dimitrie C. Popescu, *Senior Member, IEEE*, Danda B. Rawat, *Student Member, IEEE*, Otilia Popescu, *Member, IEEE*, and Mohamad Saquib, *Senior Member, IEEE*

***Abstract*—Game theory has emerged as a new mathematical tool in the analysis and design of wireless communication systems, being particularly useful in studying the interactions among adaptive transmitters that attempt to achieve specific objectives without cooperation. In this paper, we present a game-theoretic approach to the problem of joint transmitter adaptation and power control in wireless systems, where users' transmissions are subject to quality-of-service requirements specified in terms of target signal-to-interference-plus-noise ratios (SINRs) and nonideal vector channels between transmitters and receivers are explicitly considered. Our approach is based on application of separable games, which are a specific class of noncooperative games where the players' cost is a separable function of their strategic choices. We formally state a joint codeword and power adaptation game, which is separable, and we study its properties in terms of its subgames, namely, the codeword adaptation subgame and the power adaptation subgame. We investigate the necessary conditions for an optimal Nash equilibrium and show that this corresponds to an ensemble of user codewords and powers, which maximizes the sum capacity of the corresponding multiaccess vector channel model, and for which the specified target SINRs are achieved with minimum transmitted power.**

***Index Terms*—Code-division multiple access (CDMA), codeword adaptation, interference avoidance (IA), noncooperative games, power control, separable utility.**

I. Introduction

An IMPORTANT characteristic of future generations of wireless systems is their adaptation capability. Adaptive systems achieve a more efficient use of the frequency spectrum and minimize interference among systems that use the same frequency bands [1]. System adaptability is enabled by the emergence of cognitive radios that allow versatile transmitters to vary their waveforms and versatile receivers to vary their filters over time. As a consequence, the design paradigm for wireless communication systems is shifting, and traditional approaches based on fixed transmitters and heavy signal processing at the receiver are changing to new approaches that involve adaptive transmitters and receivers.

In this context, transmitter adaptation methods have become of great importance, and numerous transmitter optimization algorithms have been proposed in recent years. The main idea behind these algorithms is to optimize the transmitted signals according to the patterns of interference in the operating environment such that some specific criterion is optimized. This may be a system-wide performance measure like the total weighted squared correlation or the information-theoretic sum capacity, or it can be an individual measure such as the signal-to-interference-plus-noise ratio (SINR) or the mean squared error that corresponds to a given transmitted signal at the receiver [2]. Depending on how transmitter optimization is performed, the algorithms may be centralized [3]–[5] or distributed [2], [6]–[11]. In centralized algorithms, optimization is performed at a central location (usually the base station), which has knowledge of all transmitted signals and which assigns optimal parameters to all transmitters communicating with it. By contrast, in distributed algorithms, the transmitters adapt their signals through an iterative process based only on knowledge of the system covariance information at the base station receiver obtained through a feedback channel [12]. To provide additional flexibility and ensure that specified QoS constraints are achieved, transmitter adaptation was also combined with power control mechanisms [13], [15]. In this case, a desired QoS is specified through target SINR values, and the transmitted signal and power are jointly optimized subject to the specified QoS constraints.

In multiuser wireless systems, the transmitters may be regarded as players in a noncooperative game that choose their strategies to optimize their payoff or cost functions that are defined along specified QoS measures. We note that game-theoretic models have recently emerged as a powerful mathematical tool for the analysis and design of modern communication systems [16]. As noted in [16], some of the first game-theoretic models of communication systems have been used in the design of power control in wireless systems [17], [18]. More recently, game-theoretic approaches have been used to study codeword adaptation in code-division multiple-access (CDMA) systems [19]–[21] and joint CDMA codeword and power adaptation [22]–[24], as well as for analyzing radio resource management [25], [26] and optimal transmission strategies [27], [28].

In this paper, we present application of separable game theory to joint transmitter adaptation and power control in wireless systems, where users' transmissions are subject to QoS requirements and nonideal channels between transmitters and receivers are explicitly considered. Separable games are a

Manuscript received December 22, 2008; revised June 11, 2009. First published November 10, 2009; current version published June 16, 2010. This paper was presented in part at the 2008 IEEE Global Telecommunications Conference (GLOBECOM'08). This paper was recommended by Associate Editor T. Vasilakos.

D. C. Popescu, D. B. Rawat, and O. Popescu are with the Department of Electrical and Computer Engineering, Old Dominion University, Norfolk, VA 23529 USA (e-mail: dpopescu@odu.edu; drawa001@odu.edu; opopescu@odu.edu).

M. Saquib is with the Department of Electrical Engineering, The University of Texas at Dallas, Richardson, TX 75083-0688 USA (e-mail: saquib@utdallas.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSMCB.2009.2033704

specific class of noncooperative games where the players' cost is a separable function of their strategic choices. Although separable games have been discussed in the game-theory literature since the early days of game theory [29, Ch. 11], their application to wireless systems has only recently been considered for joint transmitter optimization and power control in the context of ideal wireless channels [22], [23]. Our work extends the game-theoretic approach in [23] and discusses application of separable games to the joint codeword and power adaptation in uplink CDMA systems with QoS constraints and nonideal vector channels between transmitters and receivers.

This paper is organized as follows: In Section II, we introduce the system model and formally state the problem studied in this paper. In Section III, we formulate joint transmitter adaptation and power control in uplink CDMA systems with nonideal vector channels and QoS requirements as a noncooperative game with separable cost functions—JCPAG. In addition, in Section III, we study the two corresponding subgames of the JCPAG—the codeword adaptation subgame (CASG) and the power adaptation subgame (PASG)—and investigate existence of Nash equilibria and conditions for optimality. In Section IV, we present an incremental algorithm that reaches the optimal Nash equilibrium of the JCPAG and study its convergence speed for various scenarios. Final discussions and conclusions are presented in Section V.

II. System Model and Problem Statement

We consider the uplink of a CDMA wireless communication system with K active users in a signal space of dimension N, where nonideal dispersive channels between users and the base station are explicitly considered. The N-dimensional received signal vector at the base station corresponding to one signaling interval is given by the expression

$$\mathbf{r} = \sum_{k=1}^{K} b_k \sqrt{p_k}\, \mathbf{H}_k \mathbf{s}_k + \mathbf{n} \tag{1}$$

where $\{\mathbf{s}_1, \ldots, \mathbf{s}_k, \ldots, \mathbf{s}_K\}$ are the N-dimensional unit-norm user codewords, $\{p_1, \ldots, p_k, \ldots, p_K\}$ are their corresponding transmitted powers, $\{b_1, \ldots, b_k, \ldots, b_K\}$ are the information symbols transmitted by users, and $\mathbf{n}$ is the additive white Gaussian noise that corrupts the received signal with zero-mean and positive definite covariance matrix $\mathbf{W} = E[\mathbf{n}\mathbf{n}^\top]$. The channels between users and the base station are described by the $N \times N$ channel matrices $\mathbf{H}_1, \ldots, \mathbf{H}_k, \ldots, \mathbf{H}_K$ assumed invertible and known at the receiver, as well as fixed for the entire duration of the transmission.

To decode the information transmitted by a given user k, the receiver uses an "inverse-channel" observation obtained by equalizing the received signal with the given user k channel matrix as in [8]

$$\begin{aligned}\mathbf{r}_k &= \mathbf{H}_k^{-1}\mathbf{r} \\ &= \underbrace{b_k \sqrt{p_k}\, \mathbf{s}_k}_{\text{desired signal}} + \overbrace{\mathbf{H}_k^{-1}\left(\sum_{\ell=1,\, k\neq\ell}^{K} b_\ell \sqrt{p_\ell}\, \mathbf{H}_\ell \mathbf{s}_\ell + \mathbf{n}\right)}^{\text{interference+noise}}. \end{aligned} \tag{2}$$

Following equalization, a matched filter is used to obtain the decision variable d_k for user k, i.e.,

$$\begin{aligned}d_k &= \mathbf{s}_k^\top \mathbf{r}_k \\ &= b_k\sqrt{p_k} + \mathbf{s}_k^\top \mathbf{H}_k^{-1}\left(\sum_{\ell=1,\ell\neq k}^{K} b_\ell \sqrt{p_\ell}\, \mathbf{H}_\ell \mathbf{s}_\ell + \mathbf{n}\right)\end{aligned} \tag{3}$$

which implies that the SINR corresponding to the symbol transmitted by user k is given by

$$\begin{aligned}\gamma_k &= \frac{p_k}{\underbrace{\mathbf{s}_k^\top \mathbf{H}_k^{-1}\left(\sum_{\ell=1,\ell\neq k}^{K} p_\ell \mathbf{s}_\ell \mathbf{H}_\ell \mathbf{H}_\ell^\top \mathbf{s}_\ell^\top + \mathbf{W}\right)\mathbf{H}_k^{-\top}\, \mathbf{s}_k}_{\mathbf{R}_k}} \\ &= \frac{p_k}{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k}.\end{aligned} \tag{4}$$

Matrix $\mathbf{R}_k$ in the denominator of the SINR expression (4) is the correlation matrix of the interference-plus-noise that affects user k's symbol in the inverse-channel observation and is related to the correlation matrix of the received signal in (1). Thus

$$\mathbf{R} = \sum_{\ell=1}^{K} p_\ell \mathbf{H}_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \tag{5}$$

by the expression

$$\begin{aligned}\mathbf{R}_k &= \mathbf{H}_k^{-1}\left(\mathbf{R} - p_k \mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{H}_k^\top\right)\mathbf{H}_k^{-\top} \\ &= \mathbf{H}_k^{-1}\mathbf{R}\mathbf{H}_k^{-\top} - p_k \mathbf{s}_k \mathbf{s}_k^\top.\end{aligned} \tag{6}$$

The denominator term in (4) is formally defined as the user interference function

$$i_k = \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \tag{7}$$

and represents the effective interference-plus-noise power that is present in user k's decision variable. For a given user k, the interference function i_k depends explicitly on the user codeword $\mathbf{s}_k$ and implicitly on all the other users codewords and powers $\mathbf{s}_\ell, p_\ell, \forall\, \ell \neq k$, but does not depend on user k's power. Similar interference functions have been defined in previous work on power control for wireless systems [30], as well as on joint power and CDMA codeword adaptation [19], [22].

In this setup, individual users may adjust their codewords and powers to meet a set of specified target SINRs $\{\gamma_1^*, \ldots, \gamma_k^*, \ldots, \gamma_K^*\}$ with minimum transmitted power. The target SINRs must be admissible and satisfy [13]

$$\sum_{k=1}^{K} \frac{\gamma_k^*}{1+\gamma_k^*} < N. \tag{8}$$

Our goal in this paper is to cast this joint codeword and power adaptation problem as a noncooperative game with separable cost functions, to investigate the existence and properties of Nash equilibrium solutions for the game and to study convergence to an optimal Nash equilibrium for the system.

III. Joint Codeword and Power Adaptation as a Noncooperative Separable Game

A noncooperative game is formally defined by a set of players, a set of strategies (or actions) associated with each player, and an individual player cost function [31]. The game is noncooperative in the sense that a given player is interested only in minimization of its individual cost function, without paying attention to how its actions affect the other players. In addition, when the user cost function is separable in the strategic choices of the given user, the game is called separable [29, Ch. 11].

For the uplink CDMA scenario in Section II, the players are the active users in the system, and their corresponding strategies consist of adaptation of their codewords and powers with strategy spaces formally defined by the N-dimensional sphere with radius 1 for the codeword strategies, i.e.,

$$\mathcal{S}_k = \left\{ \mathbf{s}_k \,|\, \mathbf{s}_k \in \mathbb{R}^N, \|\mathbf{s}_k\| = 1 \right\} \qquad \forall\, k = 1, \ldots, K \tag{9}$$

and by the real interval $(0, P_{\text{sup}}]$ (where P_{sup} is the maximum power allowed for transmission) for the power strategies

$$\mathcal{P}_k = \{ p_k \,|\, p_k \in (0, P_{\text{sup}}] \} \qquad \forall\, k = 1, \ldots, K. \tag{10}$$

Similar to [23], the cost function of a given user k is taken to be the product between the user power and its corresponding interference function, i.e.,

$$u_k = p_k \times i_k = \underbrace{p_k}_{f_k(p_k)} \times \underbrace{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k}_{g_k(\mathbf{s}_k)} \qquad \forall\, k = 1, \ldots, K. \tag{11}$$

This is a separable function with respect to the two parameters that define the user strategy choices—the corresponding codeword and power—as it can be written as a product of two functions that depend on either the power strategy $f_k(p_k)$ or the codeword strategy $g_k(\mathbf{s}_k)$.

The JCPAG is formally defined as

$$\text{JCPAG} = \left\langle \mathcal{K}, \{\mathcal{S}_k \times \mathcal{P}_k\}_{k \in \mathcal{K}}, \{u_k(\cdot)\}_{k \in \mathcal{K}} \right\rangle \tag{12}$$

where the components of the game are given in the list that follows.

1) $\mathcal{K} = \{1, \ldots, K\}$ is the set of players, which are the active users in the system.
2) $\mathcal{S}_k$ is the set of codeword strategies for player k in (9).
3) $\mathcal{P}_k$ is the set of power strategies for player k in (10).
4) $u_k : \mathcal{S} \times \mathcal{P} \longrightarrow (0, \infty)$ is the user cost function that maps the joint strategy spaces $\mathcal{S} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_K$ and $\mathcal{P} = \mathcal{P}_1 \times \cdots \times \mathcal{P}_K$ to the set of positive real numbers.

Since the user cost function is separable with respect to the power and codeword strategies, the JCPAG can be written in terms of two distinct subgames, in which individual users independently choose their codeword and power strategies to minimize their corresponding cost functions.

A. Codeword Adaptation Subgame

In this subgame, user powers are fixed, and the game is formally defined as

$$\text{CASG} = \left\langle \mathcal{K}, \{\mathcal{S}_k\}_{k \in \mathcal{K}}, \{u_k(\cdot)\}_{k \in \mathcal{K}} \right\rangle \tag{13}$$

such that individual users may adjust only their codeword strategies to minimize their cost function for the given set of powers subject to unit-norm constraints on codewords, i.e.,

$$\min_{\mathbf{s}_k} u_k \Big|_{\{p_1, \ldots, p_k\} = \text{fixed}} \quad \text{subject to} \quad \mathbf{s}_k^\top \mathbf{s}_k = 1 \qquad \forall\, k = 1, \ldots, K. \tag{14}$$

To investigate the existence of a Nash equilibrium for the CASG and to identify the best response strategies for players, we state the following formal definitions from game theory in the context of our problem.

Definition 1 (Nash Equilibrium for the CASG): The codeword ensemble $\{\mathbf{s}_1, \ldots, \mathbf{s}_{k-1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \ldots, \mathbf{s}_K\}$ is a Nash equilibrium of the CASG if, for every user $k \in \mathcal{K}$, we have that

$$u_k(\mathbf{s}_1, \ldots, \mathbf{s}_{k-1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \ldots, \mathbf{s}_K) \le u_k\left(\mathbf{s}_1, \ldots, \mathbf{s}_{k-1}, \mathbf{s}'_k, \mathbf{s}_{k+1}, \ldots, \mathbf{s}_K\right) \qquad \forall\, \mathbf{s}'_k \in \mathcal{S}_k.$$

Definition 2 (Best Response for the CASG): The best response function of user k to the other users' strategies is the set

$$B_k^{\mathbf{s}} = \{ \mathbf{s}_k \in \mathcal{S}_k \,|\, u_k(\mathbf{s}_1, \ldots, \mathbf{s}_{k-1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \ldots, \mathbf{s}_K) \le u_k(\mathbf{s}_1, \ldots, \mathbf{s}_{k-1}, \mathbf{s}'_k, \mathbf{s}_{k+1}, \ldots, \mathbf{s}_K) \qquad \forall\, \mathbf{s}'_k \in \mathcal{S}_k \}.$$

Definition 3 (Convex Game): A game is convex for a closed, convex, and bounded joint strategy space $\mathcal{S}$ if the cost function of each user u_k is convex in $\mathbf{s}_k$ for every fixed $\mathbf{s}_\ell$, such that $\ell \neq k$.

For fixed user power, the cost function in (11) is a quadratic form in the user codeword $\mathbf{s}_k$, which implies that is twice differentiable, and differentiating it twice with respect to $\mathbf{s}_k$, we get

$$\frac{\partial^2 u_k}{\partial \mathbf{s}_k^2} = 2 p_k \mathbf{R}_k. \tag{15}$$

Matrix $\mathbf{R}_k$ is the correlation matrix of the interference-plus-noise corrupting user k's inverse-channel observation (2) and has the expression given in (6). Since $\mathbf{R}_k$ is symmetric and positive definite, it implies that the user cost function is convex and that the CASG is a convex game, which ensures that it has a Nash equilibrium [23].

The best response in terms of codeword updates is found by solving the constrained optimization problem (14), as discussed in [23], and consists of a greedy interference avoidance (IA) procedure [2], in which user k's codeword is replaced by the minimum eigenvector of matrix $\mathbf{R}_k$.[1] This choice minimizes the effective interference that affects user k's inverse-channel observation and implies that, at a Nash equilibrium, all user codewords will be the minimum eigenvectors of their corresponding interference-plus-noise matrices.

The Nash equilibrium implied by the minimum eigenvector strategy is optimal with respect to the constrained minimization

[1] That is, the eigenvector corresponding to its minimum eigenvalue.

of user k cost function if the following relationship implied by the Kuhn–Tucker conditions for constrained optimum is satisfied [23]:

$$D_k^{\mathbf{s}} = (-1)\begin{vmatrix} 2p_k\left(\mathbf{R}_k - \gamma_k^*\mathbf{I}_N\right) & 2\mathbf{s}_k \\ 2\mathbf{s}_k^\top & 0 \end{vmatrix} > 0, \qquad k = 1,\ldots,K. \tag{16}$$

B. Power Adaptation Subgame

In this subgame, the user codewords are fixed, and the game is formally defined as

$$\text{PASG} = \left\langle \mathcal{K}, \{\mathcal{P}_k\}_{k\in\mathcal{K}}, \{u_k(\cdot)\}_{k\in\mathcal{K}}\right\rangle \tag{17}$$

such that individual users may adjust only their power strategies to minimize their cost function for the given ensemble of codewords subject to constraints implied by the target SINR, i.e.,

$$\min_{p_k} u_k\Big|_{\{\mathbf{s}_1,\ldots,\mathbf{s}_k\}=\text{fixed}} \qquad \text{subject to} \quad p_k = \gamma_k^*\mathbf{s}_k^\top\mathbf{R}_k\mathbf{s}_k. \tag{18}$$

Similar to the previous section, we make some formal definitions before investigating the existence of a Nash equilibrium for the PASG.

Definition 4 (Nash Equilibrium for the PASG): The set of user powers $\{p_1,\ldots,p_{k-1},p_k,p_{k+1},\ldots,p_K\}$ is a Nash equilibrium for the PASG if, for every user $k \in \mathcal{K}$, we have that

$$u_k(p_1,\ldots,p_{k-1},p_k,p_{k+1},\ldots,p_K) \le {}_k\left(p_1,\ldots,p_{k-1},p_k',p_{k+1},\ldots,p_K\right) \qquad \forall p_k' \in \mathcal{P}_k.$$

Definition 5 (Best Response for the PASG): The best response function of user k to the other users' strategies is the set

$$B_k^p = \{p_k \in \mathcal{P}_k \,|\, u_k\left(p_1,\ldots,p_{k-1},p_k,p_{k+1},\ldots,p_K\right) \le u_k\left(p_1,\ldots,p_{k-1},p_k',p_{k+1},\ldots,p_K\right) \qquad \forall p_k' \in \mathcal{P}_k\}.$$

In this case, the user cost function is linear in p_k, which may also be regarded as a convex function. Thus, following [23], a Nash equilibrium exists, and the best response strategy, which is also optimal in this case, is to update power to match the target SINR, i.e., $p_k = i_k\gamma_k^*$, for all $k = 1,\ldots,K$.

C. Nash Equilibrium for the JCPAG

Following the approach in [23] based on the result of [22, Th. 1], we note that a Nash equilibrium solution for the JCPAG exists and is defined by the codeword ensemble $\{\mathbf{s}_1,\ldots,\mathbf{s}_K\}$ and power set $\{p_1,\ldots,p_K\}$ if and only if the former is a Nash equilibrium for the CASG and the latter is a Nash equilibrium for the PASG. Since we have shown that Nash equilibria exist for both the CASG and the PASG, this implies that a Nash equilibrium for the JCPAG also exists and is implied by the best response strategies of the CASG and the PASG. We note that the Nash equilibrium codeword ensemble is not unique and that a unitary transformation of a Nash equilibrium codeword ensemble, which preserves norms and cross correlations, will imply a new codeword ensemble, which is also a Nash equilibrium for the system. We also note that a codeword ensemble corresponding to a Nash equilibrium is optimal with respect to constrained minimization of the user cost function if the sufficient conditions in (16) are satisfied.

At an optimal Nash equilibrium all user codewords are minimum eigenvectors of their corresponding interference-plus-noise correlation matrices $\mathbf{R}_k$, i.e.,

$$\mathbf{R}_k\mathbf{s}_k = \lambda_k\mathbf{s}_k, \qquad k = 1,\ldots,K \tag{19}$$

where λ_k is the minimum eigenvalue of $\mathbf{R}_k$. At a Nash equilibrium, this can be written in terms of user k target SINR γ_k^* and equilibrium power p_k as $\lambda_k = p_k/\gamma_k^*$. Using (6), which relates $\mathbf{R}_k$ and $\mathbf{R}$, we obtain an equivalent relationship to (19) that is satisfied by the codeword and power ensemble that corresponds to an optimal Nash equilibrium, i.e.,

$$\mathbf{H}_k^{-1}\mathbf{R}\mathbf{H}_k^{-\top}\mathbf{s}_k = p_k\frac{1+\gamma_k^*}{\gamma_k^*}\mathbf{s}_k, \qquad k = 1,\ldots,K. \tag{20}$$

Equation (20) implies that, at an optimal Nash equilibrium point, user codewords are eigenvectors of matrices $\mathbf{H}_k^{-1}\mathbf{R}\mathbf{H}_k^{-\top}$ with corresponding eigenvalues $p_k(1+\gamma_k^*)/\gamma_k^*$. Using the fact that a matrix and its inverse have the same eigenvectors and the corresponding eigenvalues are reciprocal, we can further write from (20) that, at the optimal Nash equilibrium, we also have that

$$\mathbf{H}_k^\top\mathbf{R}^{-1}\mathbf{H}_k\mathbf{s}_k = \frac{1}{p_k}\frac{\gamma_k^*}{1+\gamma_k^*}\mathbf{s}_k, \qquad k = 1,\ldots,K. \tag{21}$$

As discussed in [10] in a more general context of wireless systems with multiple transmitters and receivers, (21) is satisfied by an ensemble of codewords and powers that maximizes the sum capacity of the multiaccess vector channel model corresponding to the considered uplink CDMA scenario in (1). Thus, the optimal Nash equilibrium of the JCPAG, where all user cost functions are minimized subject to the specified norm and target SINR constraints, corresponds to an optimal ensemble of user codewords and powers that maximizes the sum capacity of the multiaccess vector channel model corresponding to the uplink CDMA system (1) and for which target SINRs are achieved with minimum transmitted power. This extends the similar result obtained for uplink CDMA systems with ideal channels in [23] to the nonideal channel case.

IV. Simulations and Numerical Results

In the previous section, we established the existence of Nash equilibria for the JCPAG and discussed the properties of the optimal Nash equilibrium. We note that, since multiple Nash equilibrium points for the JPCAG are possible, direct application of the best response strategies discussed in Sections III-A and III-B is not guaranteed to converge to the optimal Nash equilibrium. Thus, we follow an approach similar to that in [23] and use an incremental algorithm to study convergence to the optimal Nash equilibrium. The algorithm is based on the following incremental codeword and power updates in the direction of the best response strategy.

1) The codeword update of user k at step n of the algorithm is

$$\mathbf{s}_k(n+1) = \frac{\mathbf{s}_k(n) + m\beta\mathbf{x}_k(n)}{\|\mathbf{s}_k(n) + m\beta\mathbf{x}_k(n)\|} \tag{22}$$

where $\mathbf{x}_k$ is the best response strategy implied by the minimum eigenvector of the corresponding interference-plus-noise correlation matrix $\mathbf{R}_k$ in (6), β is a parameter that limits how far in terms of the Euclidean distance the updated codeword can be from the old codeword, and $m = \text{sgn}[\mathbf{s}_k^\top(n)\mathbf{x}_k(n)]$.

2) The power update of user k at step n of the algorithm is

$$p_k(n+1) = (1-\mu)p_k(n) + \mu\gamma_k^* i_k(n) \tag{23}$$

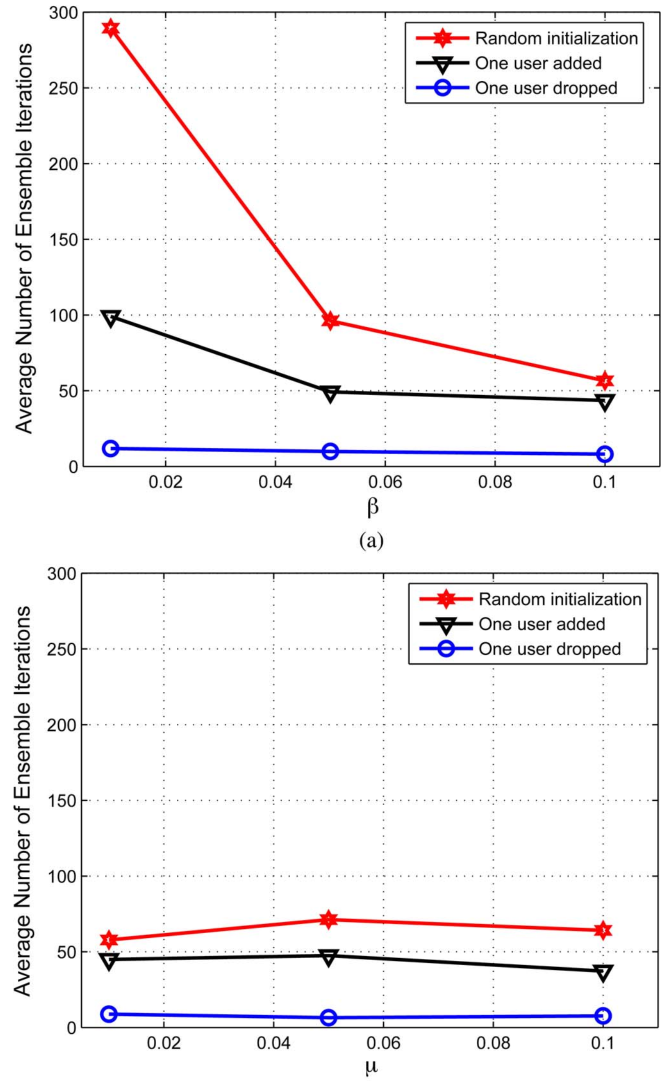
where $0 < \mu < 1$. We note that (23) is a "lagged update" in which only a fraction μ of the update is implied by the best response strategy of target SINR matching, whereas the remaining $(1-\mu)$ corresponds to the current power, and that the smaller the μ constant is, the more pronounced the lag in the power update is, and the smaller the incremental power change will be.

We formally state the algorithm here.

1) Input data:
   - Codewords $\mathbf{s}_k$, powers p_k, channel matrices $\mathbf{H}_k$, and target SINRs γ_k^* for active users $k = 1, \ldots, K$.
   - Noise covariance matrix $\mathbf{W}$.
   - Constants β, μ, and tolerance ϵ.
2) IF the admissibility condition in (8) is satisfied, GO TO Step 3. OTHERWISE, STOP. The desired system configuration is not admissible.
3) FOR each user $k = 1, \ldots, K$ DO
   a) Compute corresponding $\mathbf{R}_k(n)$ using (6) and determine its minimum eigenvector $\mathbf{x}_k(n)$.
   b) Update user k's codeword using (22).
   c) Update user k's power using (23).
4) IF the change in the cost function is larger than ϵ for any user, then GO TO Step 3. OTHERWISE, a Nash equilibrium is reached.
5) IF the optimality condition (16) is true, then STOP: an optimal Nash equilibrium has been reached. OTHERWISE, GO TO Step 3.

The check of the optimality condition (16) in Step 5 ensures that the optimal Nash equilibrium is reached and that the algorithm does not stop in a suboptimal fixed point. We note that, numerically, a fixed point of the algorithm may be reached when the codeword and power updates result in the decrease of the user cost functions that are smaller than the specified tolerance ϵ, but if the optimality condition (16) is not satisfied, the return to Step 3 and the incremental updates that will follow will move the system away from the suboptimal Nash equilibrium toward the optimal Nash equilibrium.

The proposed algorithm may be run in a centralized manner at the base station receiver or in a distributed way where individual users update codeword and power using feedback from the receiver [32], [33]. For user k to perform the updates, the interference-plus-noise correlation matrix $\mathbf{R}_k$ is needed, which can be obtained from the correlation matrix of the received signal $\mathbf{R}$ by subtracting its contribution $(p_k\mathbf{H}_k\mathbf{s}_k\mathbf{s}_k^\top\mathbf{H}_k^\top)$, as shown in (6). Thus, if the correlation matrix of the received signal $\mathbf{R}$ is made available to individual users through a feedback channel [32], [33], the proposed algorithm can work in distributed manner.



(a)

(b)

Fig. 1. Average number of ensemble iterations for convergence to the optimal Nash equilibrium of the JPCAG for $K = 6$ and $N = 5$ in 1000 trials. (a) Varying β and fixed $\mu = 0.1$. (b) Varying μ and fixed $\beta = 0.1$.

As it is the case with incremental algorithms, the convergence speed of the algorithm depends on the values of the corresponding increments specified by the algorithm constants μ and β. We performed extensive simulations of the proposed algorithm to study convergence to the optimal Nash equilibrium for the JCPAG.

In a first experiment, we studied the dependence of convergence speed on the algorithm constants μ and β for fixed values of K and N. We ran 1000 trials of the algorithm and recorded the number of ensemble iterations needed for convergence within tolerance $\epsilon = 0.001$ for $K = 6$ and $N = 5$ when starting

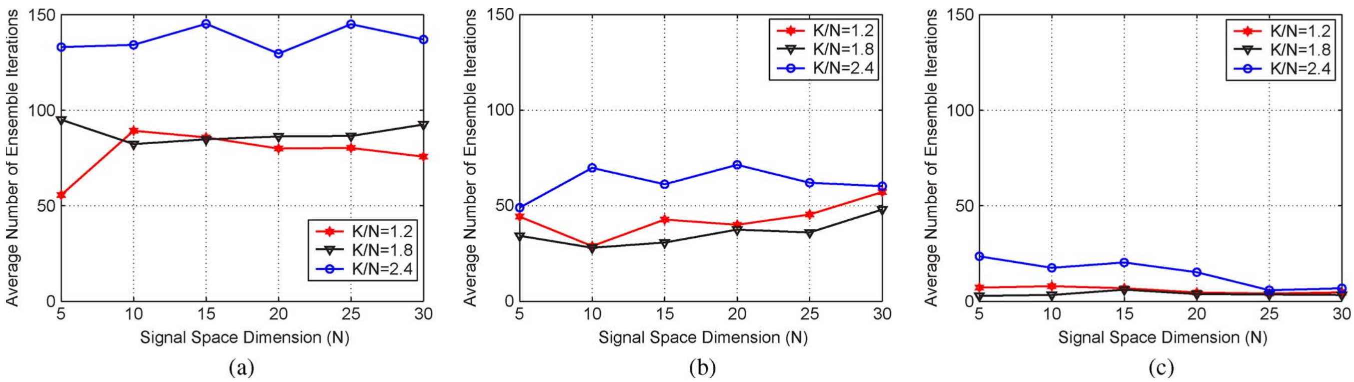


Fig. 2. Average number of ensemble iterations for convergence to the optimal Nash equilibrium of the JPCAG for fixed $\beta = 0.1$ and $\mu = 0.1$ and increasing K and N in 1000 trials. (a) Random initialization. (b) One user added to the system. (c) One user dropped from the system.

from a random ensemble of user codewords, as well as when starting from an ensemble of codewords corresponding to an optimal Nash equilibrium and adding/removing one codeword corresponding to one more/less user in the ensemble. The average number of ensemble iterations for varying β and μ is plotted in Fig. 1, from where we note that convergence to the optimal Nash equilibrium is mostly determined by the value of β and is not very sensitive to changing μ values. We also note that, for algorithm constants $\beta = 0.1$ and $\mu = 0.1$, the optimal Nash equilibrium is reached in less than 70 ensemble iterations in all considered scenarios.

In a second experiment, we looked at the convergence speed of the algorithm for increasing K and N such that their ratio K/N (also referred to as system load in CDMA systems) remains fixed. We ran 1000 trials of the algorithm with constants $\beta = 0.1$ and $\mu = 0.1$ and recorded the number of ensemble iterations needed for convergence within tolerance $\epsilon = 0.001$ for increasing N and three values of the ratio $K/N > 1$ corresponding to an overloaded system under similar scenarios as in the first experiment: starting from a random ensemble of user codewords, as well as from an ensemble of codewords corresponding to an optimal Nash equilibrium and adding/removing one codeword corresponding to one more/less user in the ensemble. Results from this experiment are shown in Fig. 2, from where we note that the average number of ensemble iterations to reach an optimal Nash equilibrium does not significantly change with increasing values of K and N: it is about the same for light and average system load ($K/N = 1.2$ and 1.8) and is only slightly higher for high system load ($K/N = 2.4$).

Additional numerical examples that illustrate the properties of the optimal Nash equilibrium in (20), as well as the tracking ability of the incremental algorithm, are discussed in [34]. These examples show that the algorithm is able to track variable target SINRs and/or variable number of active users in the system. This makes the algorithm desirable for dynamic wireless systems with users that are admitted to/dropped from the system and have variable QoS requirements.

V. Discussions and Conclusion

In this paper, we have formulated the problem of joint codeword and power adaptation in uplink CDMA systems with vector channels and QoS constraints as a noncooperative separable game—JCPAG. The game players are the active transmitters in the system, and their strategies consist of adapting corresponding CDMA codewords and transmitted powers to minimize cost functions that are separable functions of their codeword and power strategies. Our work extends the previous game-theoretic approaches to joint codeword adaptation and power control where separable games are used [22], [23] from an ideal channel scenario to a nonideal channel scenario where vector channels between transmitters and receivers are explicitly considered. The proposed approach is different from the game-theoretic approach presented in [24], which addresses a similar scenario with nonideal channels but where the utility function is different and does not have separable variables.

We have investigated the JCPAG in terms of its corresponding subgames implied by the separable variables in the considered utility function: the CASG in which players' power strategies are fixed and they update only their codeword strategies to minimize their cost function subject to unit-norm constraints on codewords, and the PASG in which players' codeword strategies are fixed and they update only their power strategies to minimize their cost function subject to QoS constraints implied by target SINRs. We have shown that the best response in terms of codeword strategies consists of greedy IA in which user k's codeword is the minimum eigenvector of the correlation matrix $\mathbf{R}_k$ of the interference-plus-noise that affects user k's inverse-channel observation, whereas in terms of power strategies, the best response is to transmit with a power that matches the desired target SINR. We have also established the existence of Nash equilibria for the JCPAG and discussed the properties of the optimal Nash equilibrium.

Because multiple Nash equilibrium points for the JPCAG exist, direct application of the best response strategies usually does not converge to the optimal Nash equilibrium, and we present an incremental algorithm similar to the one in [23] that reaches the optimal Nash equilibrium. The proposed algorithm is able to track variable target SINRs and/or variable number of active users in the system, which is a desirable feature for dynamic wireless systems with users that are admitted to/dropped from the system and have variable QoS requirements. Convergence of the proposed algorithm is studied through extensive simulations.

We conclude the paper by noting that the proposed approach is applicable to systems where intersymbol interference (ISI) can be neglected. Future work will address the issue of ISI by considering block transmission of symbols, as discussed in [8], [27], [28] where the length/duration of the block of transmitted symbols can be adjusted to ensure that ISI becomes irrelevant and can always be neglected.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their constructive comments on this paper.

REFERENCES

[1] C. Cordeiro, B. Daneshrad, J. Evans, N. Mandayam, P. Marshall, S. N. Shankar, and L. Cimini, "Guest editorial on adaptive, spectrum agile and cognitive wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 513–516, Apr. 2007.

[2] D. C. Popescu and C. Rose, *Interference Avoidance Methods for Wireless Systems*. New York: Kluwer, 2004.

[3] P. Viswanath and V. Anantharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 1984–1991, Sep. 1999.

[4] G. N. Karystinos and D. A. Pados, "New bounds on the total squared correlation and optimum design of DS-CDMA binary signature sets," *IEEE Trans. Commun.*, vol. 51, no. 1, pp. 48–51, Jan. 2003.

[5] J. A. Tropp, I. S. Dhillon, and R. W. Heath, "Finite-step algorithms for constructing optimal CDMA signature sequences," *IEEE Trans. Inf. Theory*, vol. 50, no. 11, pp. 2916–2921, Nov. 2004.

[6] S. Ulukus and R. Yates, "Iterative construction of optimum signature sequence sets in synchronous CDMA systems," *IEEE Trans. Inf. Theory*, vol. 47, no. 5, pp. 1989–1998, Jul. 2001.

[7] C. Rose, S. Ulukus, and R. Yates, "Wireless systems and interference avoidance," *IEEE Trans. Wireless Commun.*, vol. 1, no. 3, pp. 415–428, Jul. 2002.

[8] D. C. Popescu and C. Rose, "Codeword optimization for uplink CDMA dispersive channels," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1563–1574, Jul. 2005.

[9] D. C. Popescu, O. Popescu, and C. Rose, "Interference avoidance and multiaccess vector channels," *IEEE Trans. Commun.*, vol. 55, no. 8, pp. 1466–1471, Aug. 2007.

[10] O. Popescu and C. Rose, "Greedy SINR maximization in collaborative multi-base wireless systems," *EURASIP J. Wireless Commun. Netw.—Special Issue on Multiuser MIMO Networks*, vol. 2004, no. 2, pp. 201–209, Dec. 2004.

[11] G. S. Rajappan and M. L. Honig, "Signature sequence adaptation for DS-CDMA with multipath," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 2, pp. 384–395, Feb. 2002.

[12] D. J. Love, R. W. Heath, Jr., V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[13] P. Viswanath, V. Anantharam, and D. Tse, "Optimal sequences, power control and capacity of spread spectrum systems with multiuser linear receivers," *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 1968–1983, Sep. 1999.

[14] T. Guess, "Optimal sequences for CDMA with decision-feedback receivers," *IEEE Trans. Inf. Theory*, vol. 49, no. 4, pp. 886–900, Apr. 2003.

[15] D. C. Popescu and C. Rose, "Interference avoidance and power control for uplink CDMA systems," in *Proc. 58th IEEE VTC—Fall*, Orlando, FL, Oct. 2003, vol. 3, pp. 1473–1477.

[16] A. B. MacKenzie and S. B. Wicker, "Game theory in communications: Motivation, explanation, and application to power control," in *Proc. IEEE GLOBECOM*, San Antonio, TX, Nov. 2001, vol. 2, pp. 821–826.

[17] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[18] W. Yu, G. Ginis, and J. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 5, pp. 1105–1115, Jun. 2002.

[19] C. W. Sung and K. K. Leung, "On the stability of distributed sequence adaptation for cellular asynchronous DS-CDMA systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1828–1831, Jul. 2003.

[20] J. E. Hicks, A. B. MacKenzie, J. A. Neel, and J. H. Reed, "A game theory perspective on interference avoidance," in *Proc. IEEE GLOBECOM*, Dallas, TX, Dec. 2004, vol. 1, pp. 257–261.

[21] R. Menon, A. B. MacKenzie, R. M. Buehrer, and J. H. Reed, "A game theoretic framework for interference avoidance in ad-hoc networks," in *Proc. IEEE GLOBECOM*, San Francisco, CA, Dec. 2006, pp. 1–6.

[22] C. W. Sung, K. W. Shum, and K. K. Leung, "Stability of distributed power and signature sequence control for CDMA systems—A game-theoretic framework," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1775–1780, Apr. 2006.

[23] C. Lăcătuş and D. C. Popescu, "Adaptive interference avoidance for dynamic wireless systems: A game-theoretic approach," *IEEE J. Sel. Topics Signal Process.—Special Issue on Adaptive Waveform Design for Agile Sensing and Communications*, vol. 1, no. 1, pp. 189–202, Jun. 2007.

[24] S. Buzzi and H. V. Poor, "Joint receiver and transmitter optimization for energy-efficient CDMA communications," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 3, pp. 459–472, Apr. 2008.

[25] D. Niyato and E. Hossain, "Radio resource management games in wireless networks: An approach to bandwidth allocation and admission control for polling service in IEEE 802.16," *IEEE Wireless Commun.*, vol. 14, no. 1, pp. 27–35, Feb. 2007.

[26] D. Niyato and E. Hossain, "A noncooperative game-theoretic framework for radio resource management in 4G heterogeneous wireless access networks," *IEEE Trans. Mobile Comput.*, vol. 7, no. 3, pp. 332–345, Mar. 2008.

[27] G. Scutari, D. P. Palomar, and S. Barbarossa, "Optimal linear precoding strategies for wideband noncooperative systems based on game theory—Part I: Nash equilibria," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 1230–1249, Mar. 2008.

[28] G. Scutari, D. P. Palomar, and S. Barbarossa, "Optimal linear precoding strategies for wideband noncooperative systems based on game theory—Part II: Algorithms," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 1250–1267, Mar. 2008.

[29] J. C. C. McKinsey, *Introduction to the Theory of Games*. New York: McGraw-Hill, 1952.

[30] R. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1341–1348, Sep. 1995.

[31] M. J. Osborne, *An Introduction to Game Theory*. London, U.K.: Oxford Univ. Press, 2004.

[32] W. Santipach and M. L. Honig, "Signature optimization for CDMA with limited feedback," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3475–3492, Oct. 2005.

[33] D. J. Love, R. W. Heath, Jr., W. Santipach, and M. L. Honig, "What is the value of limited feedback for MIMO channels?" *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 54–59, Oct. 2004.

[34] D. B. Rawat and D. C. Popescu, "Joint codeword and power adaptation for CDMA systems with multipath and QoS requirements," in *Proc. IEEE GLOBECOM*, New Orleans, LA, Dec. 2008, pp. 4050–4054.

Dimitrie C. Popescu (S'98–M'02–SM'05) received the Engineering diploma and M.S. degree in electrical engineering from the Polytechnic Institute of Bucharest, Bucharest, Romania, in 1991 and the Ph.D. degree in electrical engineering from Rutgers University, New Brunswick, NJ, in 2002.

Between 2002 and 2006, he was with the Department of Electrical and Computer Engineering, The University of Texas at San Antonio. He has also worked for AT&T Laboratories, Florham Park, NJ, on signal processing algorithms for speech enhancement and for Telcordia Technologies, Red Bank, NJ, on wideband CDMA systems. He is currently an Assistant Professor with the Department of Electrical and Computer Engineering, Old Dominion University, Norfolk, VA. He is the coauthor of a monograph book *Interference Avoidance Methods for Wireless Systems* (Kluwer, 2004). His research interests are in the areas of wireless communications, digital signal processing, and control theory.

Dr. Popescu has served as the Technical Program Chair for the vehicular communications track of the IEEE Vehicular Technology Conference (VTC) 2009 Fall, the Finance Chair for the IEEE Multiconference on Systems and Control (MSC) 2008, and a Technical Program Committee Member for the IEEE Global Telecommunications Conference (GLOBECOM), the IEEE Wireless Communication Networking Conference (WCNC) 2006, and VTC conferences. He was the recipient of the Second Prize Award at the AT&T Student Research Symposium in 1999 for his work on interference avoidance and dispersive channels.

Danda B. Rawat (S'07) received the B.E. degree in computer engineering and the M.E. degree in information and communication engineering from Tribhuvan University, Kathmandu, Nepal, in 2002 and 2005, respectively. He is currently working toward the Ph.D. degree with the Department of Electrical and Computer Engineering, Old Dominion University, Norfolk, VA.

His research interests are in the areas of wireless communications and wireless cellular/ad hoc networks.

Otilia Popescu (S'03–M'05) received the Engineering diploma and M.S. degree in electrical engineering from the Politehnica University of Bucharest (formerly Polytechnic Institute of Bucharest), Bucharest, Romania, in 1991 and the Ph.D. degree in electrical engineering from Rutgers University, New Brunswick, NJ, in 2004.

She is currently an Adjunct Professor with the Department of Electrical and Computer Engineering, Old Dominion University, Norfolk, VA, and has worked in the past for The University of Texas at Dallas, The University of Texas at San Antonio, Rutgers University, and Politehnica University of Bucharest. Her research interests are in the general areas of communication systems, control theory, and signal processing.

Dr. Popescu has been a member of the Technical Program Committee for the IEEE Vehicular Technology Conference (VTC) 2009 Fall, the IEEE Global Telecommunications Conference (GLOBECOM) 2006, and the IEEE International Workshop on Computer-Aided Modeling, Analysis and Design of Communication Links and Networks (CAMAD) 2006.

Mohammad Saquib (M'98–SM'09) received the B.Sc. degree in electrical and electronics engineering from Bangladesh University of Engineering and Technology, Dhaka, Bangladesh, in 1991 and the M.S. and Ph.D. degrees in electrical engineering from Rutgers University, New Brunswick, NJ, in 1995 and 1998, respectively.

From 1991 to 1992, he was a System Analyst with the Energy Research Corporation, Danbury, CT. He was a Graduate Research Assistant with the Wireless Information Networks Laboratory (WINLAB), Rutgers University. From 1998 to 1999, he was a Member of Technical Staff with the Lincoln Laboratory, Massachusetts Institute of Technology (MIT), Lexington. In January 1999, he joined the Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA, where he was the Donald Ceil & Elaine T. Delaune Endowed Assistant Professor. Since July 2000, he has been with the Department of Electrical Engineering, The University of Texas at Dallas, where he is currently an Associate Professor. His research interests include various aspects of wireless data transmission including system modeling and performance, signal processing, and radio resource management, with emphasis on open-access techniques for spectrum sharing. His research interests also include designing signal processing techniques for low-cost radar and medical applications.

Prof. Saquib is an Associate Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and also served on the Editorial Board of the IEEE COMMUNICATIONS LETTERS from 2002 to 2008. He was the recipient of the Best Teaching Award "for excellence in teaching electrical engineering and telecommunications classes" from the Dean of the School of Engineering, The University of Texas at Dallas, in 2002–2003. He was a corecipient of the Best Paper Award for the paper "A robust interconnect mechanism for nanometer VLSI" at the 2007 International Test Synthesis Workshop (ITSW) and the Best Student Paper Award for the paper "Signal direction finding with low complexity" at the 2009 IEEE International Waveform Diversity and Design Conference.