A Game-Theoretic Approach to Joint Rate and Power Control for Uplink CDMA Communications

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***Abstract*—Next generation wireless systems will be required to support heterogeneous services with different transmission rates that include real time multimedia transmissions, as well as non-real time data transmissions. In order to provide such flexible transmission rates, efficient use of system resources in next generation systems will require control of both data transmission rate and power for mobile terminals. In this paper we formulate the problem of joint transmission rate and power control for the uplink of a single cell CDMA system as a non-cooperative game. We assume that the utility function depends on both transmission rates and powers and show the existence of Nash equilibrium in the non-cooperative joint transmission rate and power control game (NRPG). We include numerical results obtained from simulations that compare the proposed algorithm with a similar one which is also based on game theory and it also updates the transmission rates and powers simultaneously in a single step.**

***Index Terms*—Power control, rate control, non-cooperative games, Nash equilibrium.**

I. INTRODUCTION

NEXT generation wireless systems will provide a wide range of services for mobile users, from multimedia transmissions performed in real time to transmission of data that can tolerate delay and which is not performed in real time. In order to provide these heterogeneous services, efficient use of system resources in this case requires control of both transmission data rate and power. This requirement implies the need for joint rate and power control for mobile terminals, as opposed to only power control that is currently performed in wireless systems [1], [3], [4], [7]–[9], [13], [14], [16].

The main goal of power control is to provide adequate quality for the signal of each mobile terminal at the receiver without causing unnecessary interference to signals transmitted by other mobile terminals. Power control helps also to extend the battery life of mobile terminals by ensuring that they transmit at the minimum power level necessary to achieve the required QoS. Game-theoretic approaches to power control [1], [3], [4], [13] describe QoS for mobile terminals using utility functions. For power control the utility function for a given terminal depends usually on both the SIR and the transmission power of the terminal. In [3], [13] the utility function depends on low-level system parameters like modulation, coding, and packet size. In [4] a different utility function which depends on the channel capacity and decouples lower layer decisions like modulation and coding is proposed.

Research in the area of joint rate and power control is emerging and several algorithms have been proposed by researchers. These can be classified as centralized algorithms, such as those in [6], [11], or distributed algorithms like the ones in [2], [5], [15]. In the category of centralized algorithms the approach in reference [6] uses a different model than the one considered in our paper and is not based on game theory, while the one in [11] uses a single utility function for the system called the "system figure of merit" and deals with a dynamic scenario where some parameters of the links change during the simulations. In the distributed algorithms category, the game theoretic approach in [15] uses a similar CDMA system model as the one considered in our paper, and the proposed algorithm for joint rate and power control updates powers and rates in a similar fashion to our proposed approach, that is jointly in a single step, which results in terminals closer to the base station achieving higher rates at lower powers while terminals farther away from the base station transmit at full power with low rates. The algorithm in [5] uses a two-layered game in which the first game determines a set of rates and the second game determines the powers. This is different than our approach where powers and rates are computed jointly in a single game. The game-theoretic approach in [2] solves the problem of joint rate and power allocation using discrete link adaptation. The system model in [2] is focused explicitly on GPRS technology where discrete options for the data rate are specified through specific parameters of the modulation scheme.

In our paper we consider the uplink of a single cell CDMA system and the utility function defined in [13], but assume that each user is capable of variable transmission rates in addition to transmit powers. We formulate a NRPG which determines the optimal rate of transmission and allocates the power required for transmission based on utility maximization. We discuss existence of Nash equilibria for the NRPG and present an algorithm which reaches a Nash equilibrium in a distributed manner by updating transmission rates and powers

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jointly in a single step similar to [15]. We present numerical results obtained from simulations of the NRPG algorithm which illustrate its convergence to a Nash equilibrium and compares it with the one obtained using the approach in [15]. We note that a comparison with the alternative approaches in [2], [5] is not meaningful due to the significant differences in either the system model in the case of [2] or the update procedure in the case of [5]. We also note that we do not consider the problem of any existing users leaving (or of new users entering) the network. This is an admission control problem which is beyond the scope of the paper and was not considered in any of the aforementioned game-theoretic approaches to power control [13] or joint rate and power control [2], [5], [15].

The paper is organized as follows: in Section II, we describe the system model, and we introduce the utility function to be used for joint rate and power control. In Section III, we formulate joint rate and power control as a non-cooperative game (NRPG), and we investigate the existence of a Nash equilibrium solution for this game. In Section IV, we formally state the joint rate and power control algorithm and discuss its convergence. In Section V, we present numerical results obtained from simulations, and compare the proposed algorithm with the algorithm in [15]. Final conclusions are presented in Section VI.

II. System Model and Utility Function

We consider the uplink communication of a single-cell CDMA wireless system with N mobile terminals (users) that transmit data to the base station, for which the SIR corresponding to a given user j is [13]

$$\gamma_j = \frac{W}{r_j} \frac{h_j p_j}{\sum_{k=1, k \neq j}^{N} h_k p_k + \sigma^2} \qquad j = 1, \ldots, N \tag{1}$$

where W is the available bandwidth, h_j is the path gain of user j to the base station, r_j is the transmission rate, p_j is power of user j and σ^2 is the power spectral density of the additive white Gaussian noise (AWGN) which causes degradation of the the received signal at the base station. This expression assumes that users in the CDMA system are assigned pseudo-random noise (PN) sequences, and that conventional matched filter detectors are used at the receiver [13]. We note that because the available bandwidth is shared by all users, the transmission of any user in the system creates interference to the other users' transmissions, and the SIR defined in equation (1) serves as a measure for the QoS.

In order to achieve satisfactory QoS as implied by transmission with given rate and SIR values, users should transmit with the minimum power necessary to achieve these values. This will minimize energy consumption and will help increase the battery life for mobile terminals. The level of satisfaction of a given user i with SINR γ_i that transmits at rate r_i using power p_i can be quantified by using utility functions. The concept of utility (or payoff) is commonly used in microeconomics and game theory to denote the level of satisfaction of a decision-maker with specific products or services as a result of its actions [12]. In wireless systems utility functions are typically related to the user SIR, as well as to the transmission rate and power, and several utility functions have been used for power and rate control [3]–[6], [11], [13], [15].

In the case of power control for wireless data transmission utility functions must satisfy the following properties [13]:

- For fixed transmit powers, the utility increases with the increase in the SIR of the terminal.
- For fixed SIR, the utility decreases as the transmitted power increases.

For rate control utility functions must satisfy similar properties:

- For fixed transmission rates, the utility increases with the increase in the SIR of the terminal.
- For fixed SIR, the utility increases as the rate increases.

These properties of the utility functions are illustrated in the plots in Figure 1. As SIR increases, the terminal experiences low error rates and this leads to efficient utilization of system resources like the mobile terminals' battery drain [13]. The asymptotic increase in utility in the high SIR region is due to the fact that for sufficiently large SIR values, the error rate approaches zero [13]. Figure 2 (a) shows that for fixed interference, the utility increases initially as power increases and then it decreases for higher power values because the terminal consumes more power. The utility function behaves in a similar way for varying transmission rates as seen in Figure 2 (b).

In our game theoretic approach to joint rate and power control we consider the expression for the utility function of a given user j as in [13], but assume that the users have a variable rate

$$u_j(r_j, p_j) = \frac{L r_j f(\gamma_j)}{M p_j} \text{ [bits/J]} \qquad j = 1, \ldots, N \tag{2}$$

where $f(\gamma_j)$ is the efficiency function defined as

$$f(\gamma_j) = (1 - 2P_e)^M \tag{3}$$

where P_e is the bit error rate (BER), L is the number of information bits in a packet of length M. We consider Non-coherent FSK modulation for which the BER is given as $P_e = \frac{1}{2} e^{-\gamma/2}$. We note that we can express γ_j in terms of r_j and p_j as

$$\gamma_j = c_j \frac{p_j}{r_j} \tag{4}$$

where c_j does not depend on r_j and p_j, is given as

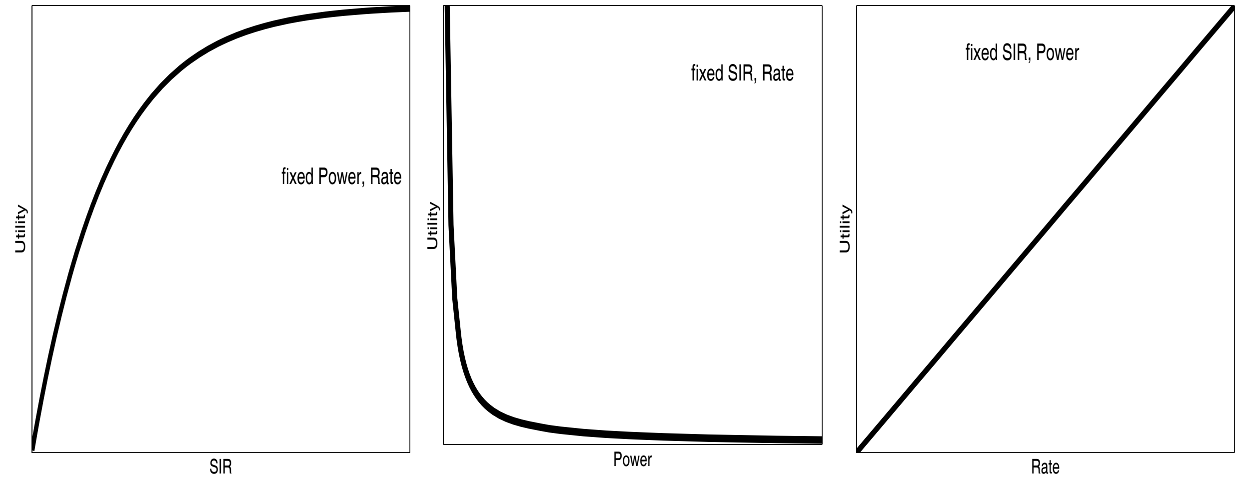
$$c_j = \frac{W h_j}{\sum_{k=1, k \neq j}^{N} h_k p_k + \sigma^2}$$

In this context, the objective of each user in the system is to adapt its transmitted power and rate in a distributed manner, such that its corresponding utility is maximized.

III. Formulation as a Non-Cooperative Game

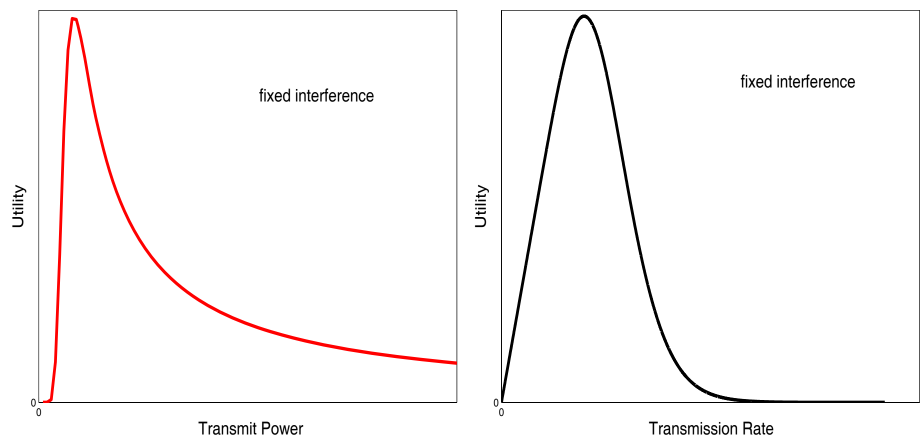
In a non-cooperative game for rate and power control each user adjusts rate r_j and power p_j in order to maximize its corresponding utility u_j. The NRPG is defined by

$$\max_{r_j \in R_j, p_j \in P_j} u_j(\mathbf{r}, \mathbf{p}) \quad j = 1, \ldots, N \tag{5}$$



(a) Utility vs. SIR (b) Utility vs. Power (c) Utility vs. Rate

Fig. 1. The behavior of utility functions for increasing SIR with fixed power and rate, increasing power with fixed SIR and rate, increasing rate with fixed power and SIR.



(a) (b)

Fig. 2. Shape of the utility as a function of the user transmission rate and power for fixed interference.

where u_j is the user utility function in (2) and R_j, P_j are the strategy spaces of user j. We note that u_j is not defined for either $r_j = 0$ or $p_j = 0$. Moreover u_j as a function of (r_j, p_j) has no limit at (0,0). So we consider strategy spaces as $R_j = (0, \overline{r}_j,]$ and $P_j = (0, \overline{p}_j,]$, which are convex sets defined in terms of minimum and maximum rates and powers. Formally we define the NRPG as $G = [\mathcal{N}, \{P_j, R_j\}, \{u_j\}]$ with $\mathcal{N} = \{1, 2, ..., N\}$ being the index set for the active users in the cell, such that each user j selects a rate $r_j \in R_j$ and a power $p_j \in P_j$ to maximize utility function u_j, where P_j represents user j's strategy set in powers and R_j represents user j's strategy set in rates. Let the rate vector $\mathbf{r} = (r_1, r_2, ..., r_N)^T \in R = R_1 \times R_2 \times ... \times R_N$, and power vector $\mathbf{p} = (p_1, p_2, ..., p_N)^T \in P = P_1 \times P_2 \times ... \times P_N$ (where T represents the transpose operator, $R_j = (0, \overline{r}_j]$ and $P_j = (0, \overline{p}_j])$ denote the outcome of the game in terms of selected rates and powers for all users.

A. The Nash Equilibrium for Individual Rate and Power Games

A Nash equilibrium is a fixed point of a non-cooperative game where no user can increase the value of its utility function through individual action. For joint rate and power control we investigate Nash equilibrium solutions for the rate and power games defined in the previous section. We will show that a Nash equilibrium solution in both transmission rates and transmit powers exists. Next, we will show that a Nash equilibrium exists in the problem of joint transmission rates and transmit powers.

First, we will consider that the players wish to maximize their utilities only with respect to transmission rates. For a given power vector $\mathbf{p}$, the utility function u_j of user j is defined on R and the game is formally represented as $G(\mathbf{p}) = [\mathcal{N}, \{R_j\}, \{u_j(., \mathbf{p})\}]$.

Then, a Nash equilibrium in rates is formally defined as

Definition 1: A rate vector $\mathbf{r} = (r_1, ..., r_N)$ is a Nash equilibrium of the NRPG $G(\mathbf{p})$ if, for every $j \in \mathcal{N}$, $u_j(r_j, \mathbf{r}_{-j}, \mathbf{p}) \geq u_j(r_{j'}, \mathbf{r}_{-j}, \mathbf{p})$ for all $r_{j'} \in R_j$, where $\mathbf{r}_{-j} = (r_1, \ldots, r_{j-1}, r_{j+1}, \ldots, r_N)^T$ is the $N-1$ dimensional vector of user rates that does not contain user j's rate.

In fact, the game $G(\mathbf{p})$ is a "dummy game", since user j's utility function depends only upon his own strategy r_j. Then a Nash equilibrium is formed by any set of N maximizing strategies of the N users. This is noteworthy because each user has only one maximizing strategy in his strategy set.

We next present two results similar to [13] that prove the existence of Nash equilibrium in the case of fixed power (rate) and variable transmission rate (power). These results are similar to [13], but they are proved in a simpler way.

Proposition 1: For each $\mathbf{p} \in P$, there is a unique maximum point of u_j in R_j. (The game $[\mathcal{N}, \{R_j\}, \{u_j(., \mathbf{p})\}]$ has a unique Nash equilibrium.)

Proof: Fixing $\mathbf{p}$, write simply $u_j(r_j)$ for $u_j(r_j, \mathbf{r}_{-j}, \mathbf{p})$. We can show that $u_j(.)$ has a unique maximizing point $\hat{r}_j$ on $(0, \infty)$ and

(a) $\hat{r}_j \in \left(0, \frac{Mp_jc_j}{2(M-1)}\right)$
(b) u_j strictly increases on $(0, \hat{r}_j)$
(c) u_j strictly decreases on $(\hat{r}_j, \infty)$

The derivative of u_j with respect to r_j is,

$$\frac{\partial u_j}{\partial r_j} = \frac{L}{Mp_j}(1 - e^{-\frac{p_jc_j}{2r_j}})^{M-1}\phi(r_j) \tag{6}$$

where

$$\phi(r_j) = 1 - e^{-\frac{p_jc_j}{2r_j}} - \frac{Mp_jc_j}{2r_j}e^{-\frac{p_jc_j}{2r_j}} \tag{7}$$

Since the first factor is positive, the sign of (6) is given by $\phi(j)$. We have

$$\frac{\partial \phi(r_j)}{\partial r_j} = \frac{1}{r_j^2}e^{-\frac{p_jc_j}{2r_j}}\left(\frac{-p_jc_j}{2} + \frac{Mp_jc_j}{2} - \frac{M^2p_j^2c_j^2}{4r_j}\right) \tag{8}$$

hence:

$$\frac{\partial \phi(r_j)}{\partial r_j}\begin{cases} < 0, & \text{if} \quad r_j \in (0, \frac{Mp_jc_j}{2(M-1)}) \\ = 0, & \text{if} \quad r_j = \frac{Mp_jc_j}{2(M-1)} \\ > 0, & \text{if} \quad r_j \in (\frac{Mp_jc_j}{2(M-1)}, \infty) \end{cases} \tag{9}$$

Since $\lim_{r_j \to 0} \phi(r_j) = 1$, and $\phi(\frac{Mp_jc_j}{2(M-1)}) = 1 - Me^{\frac{1-M}{M}} < 0$, it follows that $\phi(r_j)$ has a zero in the interval $(0, \frac{Mp_jc_j}{2(M-1)})$ (denoted by $\hat{r}_j$), and changes the sign from plus to minus at this point. Hence, we will have (a) and (b). Further, since $\lim_{r_j \to \infty} \phi(r_j) = 0$, and $\phi(r_j)$ increases on $(\frac{Mp_jc_j}{2(M-1)}, \infty)$, it follows that $\phi(r_j)$ is negative in this interval. Hence $\phi(r_j)$ is negative on $(\hat{r}_j, \infty)$, which implies (c).

It follows that the unique maximizing point of u_j in the domain R_j is r_j^* = min $\{\hat{r}_j, \overline{r}_j\} \forall j = 1, 2, ..., N$. Hence, $\mathbf{r} = (r_1^*, ..., r_N^*)$ is the unique Nash equilibrium. □

A NRPG in transmitted power is defined for each fixed $\mathbf{r} \in R$ as $G(\mathbf{r}) = [\mathcal{N}, \{P_j\}, \{u_j(\mathbf{r}, .)\}]$.

A Nash equilibrium in transmitted powers is defined formally as

Definition 2: A power vector $\mathbf{p} = (p_1, ..., p_N)$ is a Nash equilibrium of the NRPG $G(\mathbf{r})$ if for every $j \in \mathcal{N}$, $u_j(\mathbf{r}, p_j, \mathbf{p}_{-j}) \geq u_j(\mathbf{r}, p_{j'}, \mathbf{p}_{-j})$ for all $p_{j'} \in P_j$, where $\mathbf{p}_{-j} = (p_1, \ldots, p_{j-1}, p_{j+1}, \ldots, p_N)^T$ is the $N-1$ dimensional vector of user power that does not contain user j's power.

Theorem 1: For each $\mathbf{r} \in R$, the game $[\mathcal{N}, \{P_j\}, \{u_j(\mathbf{r}, .)\}]$ admits a Nash equilibrium.

Proof: For any fixed $\mathbf{r}$ and $\mathbf{p}_{-j}$, u_j has a unique maximizing point $\hat{p}_j$ on $(0, \infty)$ and

(i) $\hat{p}_j \in (0, \frac{2(M-1)r_j}{Mc_j})$
(ii) u_j strictly increases on $(0, \hat{p}_j)$
(iii) u_j strictly decreases on $(\hat{p}_j, \infty)$

Since the derivative of u_j with respect to p_j is

$$\frac{\partial u_j}{\partial p_j} = \frac{L}{Mp_j^2}(1 - e^{-\frac{p_jc_j}{2r_j}})^{M-1}\psi(p_j) \tag{10}$$

where

$$\psi(p_j) = -r_j + r_je^{\frac{-p_jc_j}{2r_j}} + \frac{Mp_jc_j}{2}e^{-\frac{p_jc_j}{2r_j}} \tag{11}$$

the proof is analogous to the proof of (a)-(c) in Proposition 1.

Now let us recall the following definition:

Definition 3: A function f is quasi-concave on a convex set D if, for every $\alpha \in \mathbb{R}$, the set $\{x \in D; f(x) \geq \alpha\}$ is convex.

From (ii) and (iii) it results that all sets $\{p_j \in (0, \infty); u_j(p_j) \geq \alpha\}$ are intervals, i.e are convex which implies that u_j is quasi concave on $(0, \infty)$.

Since the strategy spaces of the game $G(\mathbf{r})$ are not compact, we can not apply directly the equilibrium theorem (Theorem 3.1 and its Corollary) of Nikaido and Isoda [10]. *If the strategy spaces are compact convex sets in Euclidean spaces, the player i's utility function is continuous on the product space and quasi-concave with respect to the i^{th} argument, then the game has Nash equilibria.*

However, the game restricted to the strategy spaces $P_j' = [\underline{p}_j, \overline{p}_j]$, $j = 1, 2, ..., N$, admits Nash equilibria in transmission power, for every choice of $\underline{p}_j$, $0 < \underline{p}_j < \overline{p}_j$, $j = 1, 2, ..., N$, because it satisfies all assumptions of the theorem of Nikaido and Isoda [10].

Let $p_j^0 = \frac{2r_j}{\overline{c}_j}$, where $\overline{c}_j = \frac{Wh_j}{\sigma^2}$, $j = 1, 2, .., N$. Then for every j and every $\mathbf{p}_{-j}$, $B_j(\mathbf{p}_{-j}) \cap (0, p_j^0) = \emptyset$, where $B_j(\mathbf{p}_{-j})$ is the set of all best response strategies of player j against $\mathbf{p}_{-j}$. In other words, there are no equilibrium startegies of user j in the interval $(0, p_j^0)$.

To prove this assertion, pick a j and $\mathbf{p}_{-j} \in P_{-j}$. Set $\tilde{p}_j = \frac{2r_j}{c_j}$. Then $\psi(\tilde{p}_j) = r_j(-1 + e^{-1} + 2Me^{-1}) > 0$. Hence by (5), $\tilde{p}_j \in (0, \hat{p}_j)$. Then, $u_j(p_j, \mathbf{p}_{-j}) < u_j(\hat{p}_j, \mathbf{p}_{-j})$, for every $p_j \leq \tilde{p}_j$. This means that $B_j(\mathbf{p}_{-j}) \cap (0, \tilde{p}_j) = \emptyset$. Since $p_j^0 \leq \tilde{p}_j$, the above assertion is proved.

Finally, set $\underline{p}_j = \min\{p_j^0, \overline{p}_j\}$, $j = 1, 2, .., N$ and consider the restricted game with strategy spaces $[\underline{p}_j, \overline{p}_j]$. As it is shown in the above, this game admits Nash equilibria. By the last assertion, every equilibrium of the restricted game is an equilibrium of the original game. □

B. The Nash Equilibrium for the Joint Rate and Power Game

We note that the utility is a function of two variables, rate r_j and power p_j. The existence of equilibrium in transmission rates or powers does not guarantee that there is an equilibrium in the joint rate and power control game. Next we prove that there exists a Nash equilibrium in the joint rate and power control game.

Theorem 2: There exists a Nash equilibrium for the game $G = [\mathcal{N}, \{R_j \times P_j\}, \{u_j\}]$.

Proof: For each $\mathbf{p}$ define the functions v_j, $j = 1, 2, .., N$, on $(0, \infty)$ by

$$v_j(x_j) = \frac{L}{M}x_j(1 - e^{-\frac{c_j}{2x_j}})^M \tag{12}$$

We will write $v_j(x_j, \mathbf{p}_{-j})$ when we need to stress the dependence on $\mathbf{p}_{-j}$ and note that for $(\mathbf{r}, \mathbf{p}) \in R \times P$, $v_j(\frac{r_j}{p_j}, \mathbf{p}_{-j}) = u(\mathbf{r}, \mathbf{p})$.

The derivative of v_j with respect to x_j, we have

$$\frac{\partial v_j}{\partial x_j} = \frac{L}{M}(1 - e^{-\frac{c_j}{2x_j}})^{M-1}\varsigma(x_j) \tag{13}$$

where

$$\varsigma(x_j) = 1 - e^{-\frac{c_j}{2x_j}} - \frac{Mc_j}{2x_j}e^{-\frac{c_j}{2x_j}} \tag{14}$$

As above, we can show that $\lim_{x_j \to 0} \varsigma(x_j) = 1$, $\lim_{x_j \to \infty} \varsigma(x_j) = 0$ and $\frac{d\varsigma(x_j)}{dx_j}$ has a unique zero at $\frac{Mc_j}{2(M-1)}$ and changes the sign from plus to minus.

It follows that, that for each $\mathbf{p}_{-j} \in P_{-j}$, v_j has a unique maximizing point $\hat{x}_j$ on $(0, \infty)$ and

(A) $\hat{x}_j \in (0, \frac{Mc_j}{2(M-1)})$

(B) u_j strictly increases on $(0, \hat{x}_j)$

(C) u_j strictly decreases on $(\hat{x}_j, \infty)$

Now, set $\underline{x}_j = \frac{\underline{c}_j}{2M}$ and $\overline{x}_j = \frac{\overline{c}_j}{2}$, where $\underline{c}_j = \frac{Wh_j}{\sum_{k \neq j} h_k \overline{p}_k + \sigma^2}$ and $\overline{c}_j = \frac{Wh_j}{\sigma^2}$.

Then for every $\mathbf{p}$,

$$\frac{\partial v_j}{\partial x_j}(x_j, \mathbf{p}_{-j}) > 0, \quad \forall x_j \in (0, \underline{x}_j) \tag{15}$$

Therefore v_j strictly increases on $(0, \underline{x}_j)$, and

$$\frac{\partial v_j}{\partial x_j}(x_j, \mathbf{p}_{-j}) < 0, \quad \forall x_j \in (\overline{x}_j, \infty) \tag{16}$$

Therefore v_j strictly decreases on $(\overline{x}_j, \infty)$.

Indeed, we have

$$\varsigma\left(\frac{c_j}{2M}\right) = 1 - e^{-M} - M^2 e^{-M} > 0 \tag{17}$$

and

$$\varsigma\left(\frac{c_j}{2}\right) = 1 - (M+1)e^{-1} < 0 \tag{18}$$

Then, (B) and (C) imply

$$\frac{\partial v_j}{\partial x_j}(x_j, \mathbf{p}_{-j}) > 0, \quad \forall x_j \in (0, \frac{c_j}{2M}) \tag{19}$$

$$\frac{\partial v_j}{\partial x_j}(x_j, \mathbf{p}_{-j}) < 0, \quad \forall x_j \in (\frac{c_j}{2}, \infty) \tag{20}$$

respectively. Since $\underline{x}_j \leq \frac{c_j}{2M}$ and $\overline{x}_j \geq \frac{c_j}{2}$ for every $\mathbf{p}$, it follows that (15) and (16) hold.

Once again, we use the Nikaido-Isoda theorem for the restricted game $[\mathcal{N}, \{[\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j]\}, \{u_j(\cdot,\cdot)\}]$, where $0 < \underline{r}_j \leq \overline{r}_j$ and $0 < \underline{p}_j \leq \overline{p}_j$.

Obviously, the strategy spaces are nonempty, compact and convex and the utility functions are jointly continuous in (r_j, p_j).

The shape of v_j (as a function of x_j), as it results form (B) and (C) shows that this function is quasi-concave on $(0, \infty)$, more precisely, the upper contour sets of the form $\{x_j \in (0, \infty) | v_j(x_j) \geq \alpha\}$ are intervals. Now observe that, for any real number α, and for any $\mathbf{p}_{-j} \in P_{-j}$,

$$\{(r_j, p_j) \in [\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j] | u_j(r_j, p_j, p_{-j}) \geq \alpha\}$$
$$= \{(r_j, p_j) \in [\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j] | v_j(\tfrac{r_j}{p_j}, p_{-j}) \geq \alpha\}$$

Suppose that $\{x_j \in (0, \infty) | v_j(x_j) \geq \alpha\} = [a, b]$. (The cases when this set is $(0, a)$ or (b, ∞) could also be considered). Then

$$\{(r_j, p_j) \in [\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j] | v_j(\tfrac{r_j}{p_j}, p_{-j}) \geq \alpha\}$$
$$= \{(r_j, p_j) \in [\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j] | ap_j \leq r_j \leq bp_j\}$$

which is a convex set in $\mathbb{R}^2$. Hence, u_j is quasi-concave in (r_j, p_j).

We can conclude that the restricted game has a Nash equilibrium.

Finally, we will argue that the game $[\mathcal{N}, \{R_j \times P_j\}, \{u_j(\cdot,\cdot)\}]$ admits a Nash equilibrium.

For each j, choose $\underline{r}_j$, $\underline{p}_j$ such that $0 < \underline{r}_j \leq \min\{\frac{\underline{c}_j}{2M}\overline{p}_j, \overline{r}_j\}$, $0 < \underline{p}_j \leq \min\{\frac{2\overline{r}_j}{\overline{c}_j}, \overline{p}_j\}$. By the above, it results that the restricted game $[\mathcal{N}, \{[\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j]\}, \{u_j(\cdot,\cdot)\}]$ has equilibria. Let (r^*, p^*) be a Nash equilibrium of this game. Then, for every j,

$$u_j(r_j, p_j, \mathbf{p}^*_{-j}) \leq u_j(\mathbf{r}^*, \mathbf{p}^*), \quad \forall (r_j, p_j) \in [\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j] \tag{21}$$

Next, we will prove that $(\mathbf{r}^*, \mathbf{p}^*)$ is an equilibrium of the game $[\mathcal{N}, \{R_j \times P_j\}, \{u_j(\cdot,\cdot)\}]$.

Pick a j and $(r_j, p_j) \in R_j \times P_j$ and denote $x_j = \frac{r_j}{p_j}$. Three possible situations will be considered:

(I) $x_j \in [\frac{\underline{r}_j}{\overline{p}_j}, \frac{\overline{r}_j}{\underline{p}_j}]$

(II) $x_j < \frac{\underline{r}_j}{\overline{p}_j}$

(III) $x_j > \frac{\overline{r}_j}{\underline{p}_j}$

In case (I) we show that there exists $(r'_j, p'_j) \in [\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j]$ such that $x_j = \frac{r'_j}{p'_j}$.

Obviously, since $x_j \in [\frac{\underline{r}_j}{\overline{p}_j}, \frac{\overline{r}_j}{\underline{p}_j}]$, then either $x_j \in [\frac{\underline{r}_j}{\overline{p}_j}, \frac{\overline{r}_j}{\overline{p}_j}]$, or $x_j \in [\frac{\overline{r}_j}{\overline{p}_j}, \frac{\overline{r}_j}{\underline{p}_j}]$.

In the first case take $r'_j = x_j\overline{p}_j$ and $p'_j = \overline{p}_j$. ($r'_j \geq \frac{\underline{r}_j}{\overline{p}_j}\overline{p}_j = \underline{r}_j, r'_j \leq \frac{\overline{r}_j}{\overline{p}_j}\overline{p}_j = \overline{r}_j$)

In the second case take $r'_j = \overline{r}_j$ and $p'_j = \frac{\overline{r}_j}{x_j}$. ($p'_j \geq \overline{r}_j\frac{\underline{r}_j}{\overline{r}_j} = \underline{p}_j, p'_j \leq \overline{r}_j\frac{\overline{p}_j}{\overline{r}_j} = \overline{p}_j$)

Then, by (21)

$$u_j(r_j, p_j, \mathbf{p}^*_{-j}) = v_j(x_j, \mathbf{p}^*_{-j}) = v_j(\frac{r'_j}{p'_j}, \mathbf{p}^*_{-j})$$
$$= u_j(r'_j, p'_j, \mathbf{p}^*_{-j}) \leq u_j(r^*_j, p^*_j) \tag{22}$$

In case (II), $x < \frac{\underline{c}_j}{2M} = \underline{x}_j$ and by (15) and (21) we have

$$u_j(r_j, p_j, \mathbf{p}^*_{-j}) = v_j(x_j, \mathbf{p}^*_{-j}) < v_j(\frac{\underline{r}_j}{\overline{p}_j}, \mathbf{p}^*_{-j})$$
$$= u_j(\underline{r}_j, \overline{p}_j, \mathbf{p}^*_{-j}) \leq u_j(r^*_j, p^*_j) \tag{23}$$

Similarly, in case (III), since $x_j > \frac{\overline{c}_j}{2} = \overline{x}_j$, it follows from (16) and (21) that

$$u_j(r_j, p_j, \mathbf{p}^*_{-j}) = v_j(x_j, \mathbf{p}^*_{-j}) < v_j(\frac{\overline{r}_j}{\underline{p}_j}, \mathbf{p}^*_{-j})$$
$$= u_j(\overline{r}_j, \underline{p}_j, \mathbf{p}^*_{-j}) \leq u_j(r^*_j, p^*_j) \tag{24}$$

□

IV. The Joint Rate and Power Control Algorithm

In the proposed NRPG algorithm for joint rate and power control, the users update their rates and powers asynchronously such that for a given user the new rate and power values are computed in the same step. Note that, as shown in the proof of Theorem 3, for finding a Nash equilibrium of the game $[\mathcal{N}, \{R_j \times P_j\}, \{u_j(\cdot,\cdot)\}]$, it is enough to consider the restricted game $[\mathcal{N}, \{[\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j]\}, \{u_j(\cdot,\cdot)\}]$, where $\underline{r}_j$ and $\underline{p}_j$ are chosen as in Theorem 2. The joint rate and power control algorithm based on NRPG is formally stated below:

NRPG Algorithm

(1) Initialize the rates $\mathbf{r}(t_0) = [r_1(t_0), ..., r_N(t_0)]$ and power vector $\mathbf{p}(t_0) = [p_1(t_0), ..., p_N(t_0)]$ at time t_0 with $r_j(t_0) = \overline{r}_j$ and $p_j(t_0) = \underline{p}_j$, $j = 1, 2, ..., N$.

(2) Each user j ($j \in \mathcal{N}$) at time instant t_k computes:

a) Compute $x_j(t_k)$, for $t_k \geq t_0$, as

$$x_j(t_k) = \arg \max_{x_j \in [\frac{\underline{r}_j}{\overline{p}_j}, \frac{\overline{r}_j}{\underline{p}_j}]} v_j(x, \mathbf{p}_{-j}(t_k))$$

b) Evaluate the rates $r_j(t_{k+1})$ and powers $p_j(t_{k+1})$ as

$$(r_j(t_{k+1}), p_j(t_{k+1})) = \begin{cases} (x_j(t_k)\overline{p}_j, \overline{p}_j), \\ \text{if} \quad x_j(t_k) \leq \frac{\overline{r}_j}{\overline{p}_j} \text{ (I)} \\ \\ (\overline{r}_j, \frac{\overline{r}_j}{x_j(t_k)}), \\ \text{if} \quad x_j(t_k) > \frac{\overline{r}_j}{\overline{p}_j} \text{ (II)} \end{cases}$$

(3) If $\max\{\|\mathbf{r}(t_k) - \mathbf{r}(t_{k+1})\|, \|\mathbf{p}(t_k) - \mathbf{p}(t_{k+1})\|\} \leq \epsilon$, then STOP. Else, make $k = k + 1$ and go to step (2).

The power and rate update equations require the knowledge of the interference plus noise experienced by user j's signal at the base station i.e.

$$\left(\sum_{k=1, j \neq k}^{N} h_k p_k + \sigma^2 \right). \tag{25}$$

It is assumed that the base station broadcasts the "total power plus noise" term $\left(\sum_{k=1}^{N} h_k p_k + \sigma^2\right)$ to all the users. In addition, assuming reciprocal communication channels each user j knows its channel gain to the base station h_j and can estimate their received power $h_j p_j$ at the base station which is subtracted from the "total power plus noise" term to obtain the term in (25) needed for updating transmission rates and powers.

Theorem 3: For each j, the sequence $r_j(t)$ decreases and converges to a limit r_j^*, and the sequence $p_j(t)$ increases and converges to a limit p_j^*. The pair $(\mathbf{r}^*, \mathbf{p}^*)$ is a Nash equilibrium of the game.

Proof: First recall that, for each $\mathbf{p}_{-j}$, the function $v_j(\cdot, \mathbf{p}_{-j})$ has a unique maximizing point on $(0, \infty)$, denoted by $\hat{x}_j$, and this is the unique root of the equation (14). As $\hat{x}_j$ depends on $\mathbf{p}$, this dependence can be expressed as the dependence on c_j. Therefore, considering $\varsigma(x_j)$ as a function of (x_j, c_j), the equation $\varsigma(x_j, c_j) = 0$ defines implicitly $\hat{x}_j$ as a function of c_j, $\hat{x}_j = \hat{x}_j(c_j)$. The implicit function theorem says that

$$\frac{\partial \hat{x}_j}{\partial c_j} = -\frac{\frac{\partial \varsigma}{\partial c_j}}{\frac{\partial \varsigma}{\partial x_j}}$$

which by a simple calculation gives us $\frac{\partial \hat{x}_j}{\partial c_j} = \frac{x_j}{c_j} > 0$, i.e. $\hat{x}_j$ strictly increases with c_j.

Now, we can prove the first part of the theorem by induction on t_k.

It is obvious that, for each j, $r_j(t_1) \leq r_j(t_0) = \overline{r}_j$ and $p_j(t_1) \geq p_j(t_0) = \underline{p}_j$.

Now, suppose that $r_j(t_k) \leq r_j(t_{k-1})$ and $p_j(t_k) \geq p_j(t_{k-1})$ for some $k \geq 1$ and for every j. Then, for each j, $c_j(t_k) \leq c_j(t_{k-1})$ and since

$$\arg \max_{x_j \in [\frac{\underline{r}_j}{\overline{p}_j}, \frac{\overline{r}_j}{\underline{p}_j}]} v_j(x, \mathbf{p}_{-j}) = \arg \max_{x_j \in [0, \infty]} v_j(x, \mathbf{p}_{-j})$$

it follows from the above that $x_j(t_k) \leq x_j(t_{k-1})$.

This means that, in the NRPG algorithm at the Step (2), b) at time t_{k+1}, either the user switches strategy from case (II) to case (I) (step (2) b) of NRPG algorithm) or the user retains the strategy of case (II) but increases p_j and decreases r_j. In both situations $r_j(t_k) \leq r_j(t_{k-1})$ and $p_j(t_k) \geq p_j(t_{k-1})$.

Since the sequences $r_j(t_k)$ and $p_j(t_k)$ are monotone in bounded intervals, they are convergent.

Let r_j^* and p_j^*, respectively be the limits of these sequences. Now, since $x_j(t_k) = \frac{r_j(t_{k+1})}{p_j(t_{k+1})}$, it follows that

$$u_j(r_j(t_{k+1}), p_j(t_{k+1}), \mathbf{p}_{-j}(t_k)) = v_j(x_j(t_k), \mathbf{p}_{-j}(t_k))$$
$$\geq v_j(x, \mathbf{p}_{-j}(t_k)), \forall x \in \left[\frac{\underline{r}_j}{\overline{p}_j}, \frac{\overline{r}_j}{\underline{p}_j}\right]$$

Now, pick an arbitrary $(r_j, p_j) \in [\underline{r}_j, \overline{r}_j] \times [\underline{p}_j, \overline{p}_j]$. Then, $\frac{r_j}{p_j} \in \left[\frac{\underline{r}_j}{\overline{p}_j}, \frac{\overline{r}_j}{\underline{p}_j}\right]$, so that, the above inequality implies that

$$u_j(r_j(t_{k+1}), p_j(t_{k+1}), \mathbf{p}_{-j}(t_k)) \geq$$
$$v_j\left(\frac{r_j}{p_j}, \mathbf{p}_{-j}(t_k)\right) = u_j(r_j, p_j, \mathbf{p}_{-j}(t_k))$$

By continuity of u_j, the inequality between the extreme terms yields

$$u_j(r_j^*, p_j^*, \mathbf{p}_{-j}^*) \geq u_j(r_j, p_j, \mathbf{p}_{-j}^*)$$

□

Theorem 3 proves that for the choice of initial rates and power values specified in Step (1) of the NRPG algorithm this will always reach the same Nash equilibrium point characterized by the pair $(\mathbf{r}^*, \mathbf{p}^*)$ where the individual elements r_j^* in $\mathbf{r}^*$ and p_j^* in $\mathbf{p}^*$ contain the limits of the sequences $r_j(t)$ and $p_j(t)$ which are monotonically decreasing and increasing, respectively for all j. Thus, even though many Nash equilibria for the NRPG game may be possible depending on the initial strategies of the players, the NRPG algorithm with the initialization specified in Step (1) will always converge to the same Nash equilibrium point and from this perspective the outcome of the NRPG algorithm is unique. We note that, from a practical perspective, all users are interested in transmitting at highest rate possible with lowest required power, and in

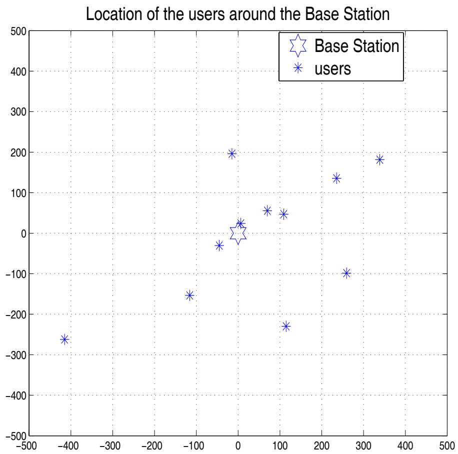


Fig. 3. Example of random locations of users around the base station.

the context of Theorem 3 initializing user powers with the minimum value in the power strategy space and user rates with the maximum value in the rate strategy space is a rational choice.

V. Simulation Setup and Numerical Results

We performed simulations to study the NRPG algorithm in a single cell CDMA system and compared it with the rate and power control algorithm described in [15] which we refer to as the 'ZL algorithm' (or simply 'ZL'). The ZL algorithm initializes all user rates and powers with zero, and in one step of the algorithm each user updates rate and power with the corresponding solutions of a nonlinear programming problem with two variables that maximize the user payoff function. A formal statement of the ZL algorithm is given in [15] and we refer readers to [15] for specific details about it.

The parameters of the simulation setup were chosen similar to [15] and these are:

- All users are assumed to be stationary, and the propagation model has channel gains $h_j = c/d_j^4$, where d_j is the distance (in meters) between user j and the base station and $c = 0.097$.
- The power spectral density of the AWGN at the receiver is $\sigma^2 = 5 \times 10^{-15}$ W/Hz.
- The minimum and maximum powers of each user are $\underline{p} = \hat{p} = 10^{-6}$ W, respectively $\overline{p} = 0.2$ W.
- The minimum and maximum transmission rates of each user are $\underline{r} = 0.1$ and $\overline{r} = 96,000$ [bits/sec] respectively.

The NRPG algorithm is compared to the ZL algorithm for different available bandwidths by considering the location of the users around the base station as shown in Figure 3. The results are shown in Figure 4. When available bandwidth is $W = 3.84 \times 10^6$ Hz as in the numerical examples in [15] the NRPG and the ZL algorithms result in similar powers and rates, with users closer to the base station transmitting at higher rates and lower powers than users farther away from the base station. For both algorithms, as the available bandwidth W is increased the number of users that transmit at the maximum allowed power $\overline{p}$ and/or minimum allowed rate $\underline{r}$ decreases, and the NRPG algorithm results in more efficient power and rate allocations than the ZL algorithm.

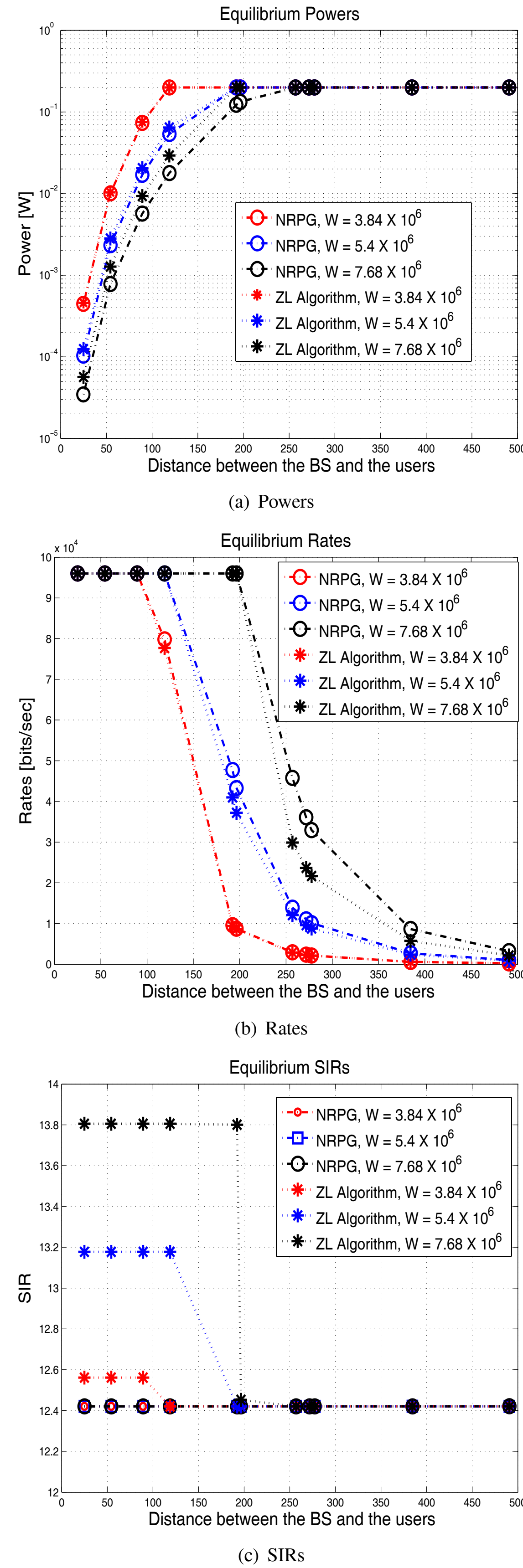
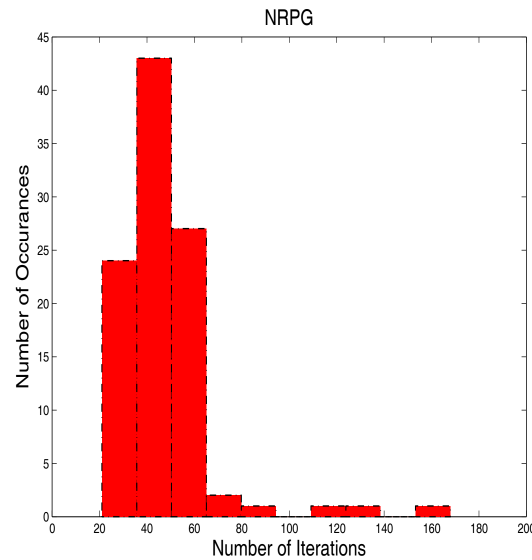


Fig. 4. Comparison of user powers, rates and SIRs yielded by the ZL algorithm ([15]) with those yielded by the NRPG for different bandwidths

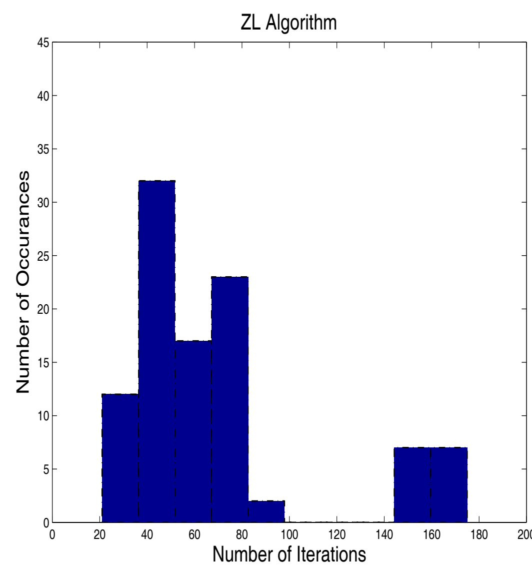
As seen in Figure 4(a), when the available bandwidth is increased by about 40% to $W = 5.4 \times 10^6$ Hz the power values for users that are transmitting below the maximum power level are about 40% less for the NRPG algorithm than for the ZL algorithm, and when the available bandwidth is doubled, to $W = 7.68 \times 10^6$ Hz, these powers for the NRPG are about half of the corresponding ZL powers. In terms of rates, shown in Figure 4(b), when the bandwidth is increased to $W = 5.4 \times 10^6$ Hz the rate values for users that are transmitting at rates between the minimum rate $\underline{r}$ and maximum rate $\overline{r}$ are approximately 10–20% higher for the NRPG algorithm than for the ZL algorithm, and when the bandwidth is $W = 7.68 \times 10^6$ Hz the rates for the NRPG are about 25% to 50% higher than the corresponding ZL rates. In addition, as seen in Figure 4(c) the NRPG algorithm results always in the equal SIR values for all users as opposed to the ZL algorithms where users that are closer to the base station get slightly higher SIR values.

In order to corroborate these results and evaluate the average improvements implied by the NRPG algorithm over the ZL algorithm we also performed Monte Carlo simulations and ran 100 trials of both algorithms, each trial with a different random placement of the users around the base station. Results of these simulations are shown in Figures 5 – 7 and confirm that the NRPG algorithm yields more efficient power and rate allocations than the ZL algorithm: when the available bandwidth is increased average power values (over the 100 simulations) for users that are transmitting below the maximum power level are lower for the NRPG algorithm than for ZL algorithm (Figure 5) and average rate values (over the 100 simulations) for users that are transmitting at rates between the minimum and maximum values are higher for the NRPG algorithm than for the ZL algorithm (Figure 6). From Figure 7, we note that the average SIR values (over the 100 simulations) are similar for the NRPG algorithm while for the ZL algorithm the average SIR values for users that are closer to the base station are slightly higher than those of the rest of the users.

Monte Carlo simulations were also used to investigate convergence speed of the NRPG and ZL algorithms, and histograms with the number of steps for convergence to Nash equilibrium in the 100 trials simulated are shown in Figure 8. We note that the NRPG algorithm takes around 20 – 60 iterations to converge to the equilibrium (Figure 8(a)) while the ZL algorithm takes 40 – 80 iterations (Figure 8(b)). Thus, in addition to the more efficient power and rate allocations noted above the NRPG algorithm is also faster than the ZL algorithm.



(a) NRPG



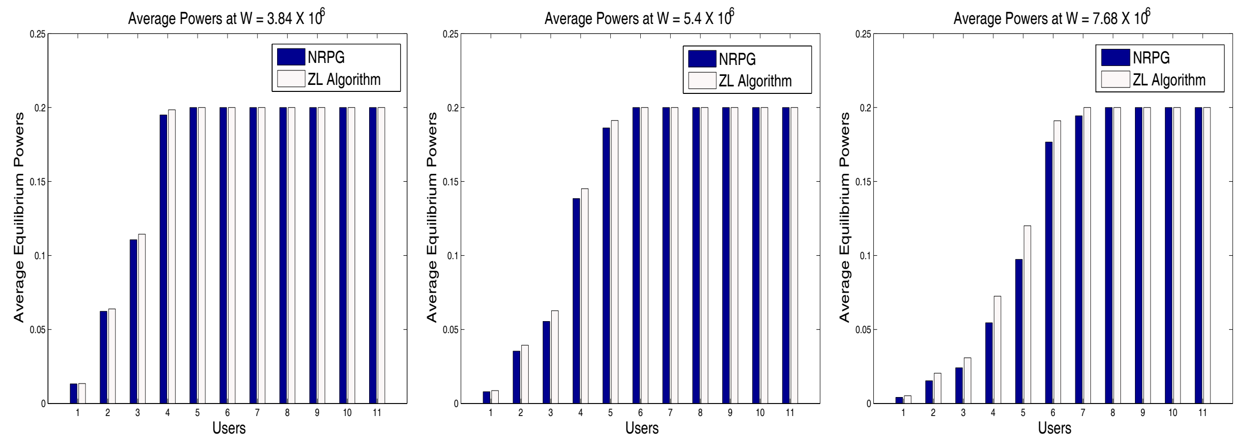
(b) ZL algorithm

Fig. 8. Monte Carlo simulations to investigate convergence of the NRPG and ZL algorithms.

VI. Conclusions and Future Work

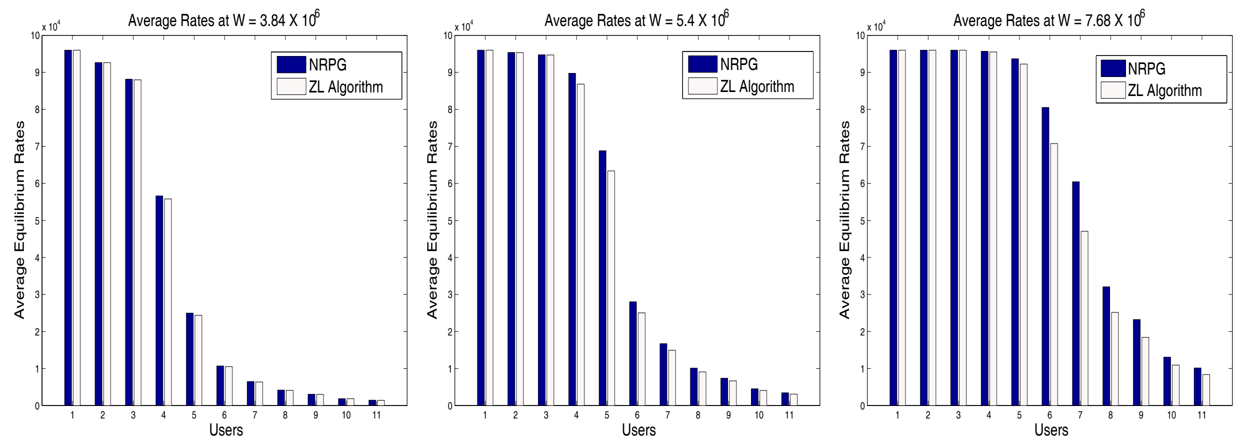
In this paper we applied game theory to model the problem of joint transmission rate and power control for the uplink of a single cell CDMA system as a non-cooperative game. Similar to other approaches to joint rate and power control a static scenario with no users leaving or new users joining the system was considered. The utility function used in our approach is defined by the ratio of throughput to the transmit power and its maximization implies optimal transmission rate and power. We showed the existence of Nash equilibria for the joint rate and power control game and formally stated the NRPG algorithm for joint rate and power control.

Numerical simulations were performed to illustrate the proposed NRPG algorithm and to compare it with the ZL algorithm for joint rate and power control developed using a related game-theoretic approach in [15]. While both algorithms work similarly and update user rates and powers simultaneously in one step, simulation results have shown that the NRPG algorithm converges faster than the ZL algorithm and results in more efficient power and rate allocations. A comparison of the Nash equilibrium point implied by the NRPG algorithm with the optimal power and rate allocations implied by centralized schemes (such as those in [6], [11] for example) will be the object of future work.



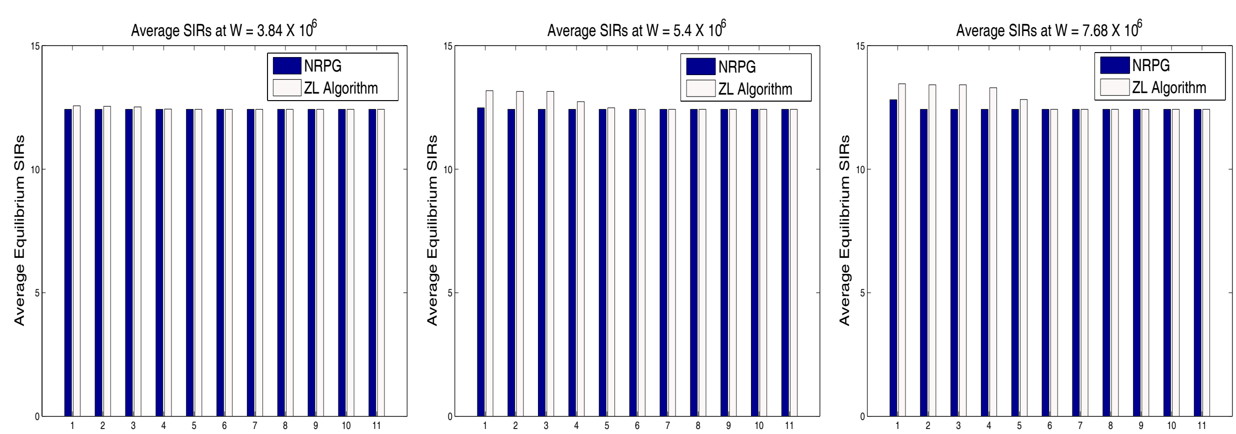
(a) W= 3.84×10^6 (b) W= 5.4×10^6 (c) W= 7.68×10^6

Fig. 5. The average powers yielded by the ZL algorithm and NRPG for a Monte Carlo simulation of 100 runs. User 1 is the closest to the base station and user 11 is the farthest.



(a) W= 3.84×10^6 (b) W= 5.4×10^6 (c) W= 7.68×10^6

Fig. 6. The average rates yielded by the ZL algorithm and NRPG for a Monte Carlo simulation of 100 runs. User 1 is the closest to the base station and user 11 is the farthest.



(a) W= 3.84×10^6 (b) W= 5.4×10^6 (c) W= 7.68×10^6

Fig. 7. The average SIRs yielded by ZL algorithm and NRPG for a Monte Carlo simulation of 100 runs. User 1 is the closest to the base station and user 11 is the farthest.

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