Joint Codeword and Power Adaptation for CDMA Systems with Multipath and QoS Requirements

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Abstract—In this paper we present an algorithm for joint codeword and power adaptation applicable in the uplink of a Code Division Multiple Access (CDMA) system in which multipath channels between users and the base station receiver are explicitly considered. The proposed algorithm uses incremental updates for codeword and power that allow a smooth transition of the system to a steady state configuration where specified targets for the signal-to-interference+noise-ratio (SINR) are achieved with minimum powers. The algorithm is illustrated with numerical results obtained from simulations.

I. INTRODUCTION

CDMA enables multiuser communications along with efficient utilization of available spectrum and transmitter power in wireless systems and has been proposed for use in future generation wireless systems. Unlike Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA) systems where transmitters are restricted to using the available spectrum part of the time, respectively part of the spectrum all of the time, in a CDMA system the transmitters can use all available spectrum all of the time, and the access to the air interface is controlled by distinct codewords (or spreading sequences) that are assigned to active users in the system [2], [21]. In general, with non-orthogonal codewords, signals transmitted at the same time by distinct transmitters interfere with each other at the receiver and create multiple access interference (MAI). In order to minimize the effects of MAI and ensure that specified Quality of Service (QoS) requirements are met, the transmitters in a CDMA system may adjust their codewords and/or powers and numerous algorithms have been proposed for codeword and power adaptation in CDMA systems lately [4], [5], [7], [8], [17], [22]–[24].

In addition to MAI, signals in a CDMA system are also affected by propagation through multipath and fading channels which further degrades their quality at the receiver, and we note that algorithms for codeword optimization for CDMA systems with multipath are discussed in [1], [3], [10], [15], [16], [19]. We also note that these algorithms focus exclusively on the optimization of CDMA codewords and do not specify QoS requirements for users in the system, and that a gradient-based algorithm for joint codeword and power adaptation in CDMA systems with multipath and QoS requirements was proposed recently [12].

In this paper we present a new algorithm that is based on extending the adaptive interference avoidance (IA) technique in [7] from the ideal channel case to the practical case where channels between users and the base station are explicitly considered. Our work is motivated by the fact that the extensive simulations of the gradient-based algorithm in [12] have shown numerous instances where this gets trapped in sub-optimal points where users distribute their energy inefficiently and are difficult to escape.

The paper is organized as follows: in Section II we present the system model and formally state the problem followed by extension of the separable game theory framework outlined in [20] from the ideal channel scenario in [7] to the multipath channel scenario. The proposed algorithm is formally stated in Section IV where its convergence to a fixed point is also discussed. We illustrate the algorithm with numerical examples obtained from simulations in Section V, and present final conclusions in Section VI.

II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider the uplink of a synchronous CDMA system with $K$ active users in a signal space of dimension $N$ where multipath channels between users and base station are explicitly considered. The $N$-dimensional received signal vector at the base station corresponding to one signaling interval is given by the expression

$$ r = \sum_{k=1}^{K} b_k \sqrt{p_k} H_k s_k + n $$

where $\{s_1, \ldots, s_K\}$ are the $N$-dimensional user codewords assumed to have unit norm, $\{p_1, \ldots, p_K\}$ are the received powers at the base station, $\{b_1 \ldots b_k \ldots b_K\}$ are the information symbols transmitted by users, and $n$ is the additive Gaussian noise that corrupts the received signal with zero-mean and positive definite covariance matrix $W = E[nn^\top]$. The multipath channels between users and the base station are described by the $N \times N$ diagonal matrices $H_1, \ldots, H_b, \ldots, H_K$ assumed invertible and known at the receiver as well as fixed for the entire duration of the transmission [15].

In order to decode the information transmitted by a given user $k$ the receiver uses an “inverse-channel” observation obtained by equalizing the received signal with the given
user $k$ channel matrix as in [15]
\[ r_k = H_k^{-1} r \]
\[ = b_k \sqrt{p_k s_k} + H_k^{-1} \left( \sum_{\ell=1, \ell \neq k}^K b_{\ell} \sqrt{p_{\ell}} r_{\ell} + n \right) \]

Following the equalization a matched filter (MF) is used to obtain the decision variable $d_k$ for user $k$
\[ d_k = s_k^T r_k \]
\[ = b_k \sqrt{p_k} s_k + s_k^T H_k^{-1} \left( \sum_{\ell=1, \ell \neq k}^K b_{\ell} \sqrt{p_{\ell}} r_{\ell} + n \right) \]  

With these notations the SINR for user $k$ is given by
\[ \gamma_k = \frac{p_k}{s_k^T H_k^{-1} \left( \sum_{\ell=1, \ell \neq k}^K p_{\ell} s_{\ell} H_{\ell}^T s_{\ell} + W \right) H_k^T s_k} \]  

where matrix $R_k$ in the denominator of the SINR expression (4) is the correlation matrix of the interference+noise that affects user $k$’s symbol in the “inverse-channel” observation and is related to the correlation matrix of the received signal in equation (1)
\[ R = \sum_{\ell=1}^K p_{\ell} H_\ell s_\ell H_\ell^T + W \]  

by the expression
\[ R_k = H_k^{-1} \left( R - p_k H_k s_k s_k^T H_k^T \right) H_k^T \]
\[ = H_k^{-1} R H_k^T - p_k s_k s_k^T \]

The expression of user $k$ SINR can then be rewritten in the simpler form
\[ \gamma_k = \frac{p_k}{s_k^T R_k s_k} \]  

where the denominator term represents the effective interference+noise power that is present in user $k$’s decision variable
\[ i_k = s_k^T R_k s_k \]

The interference function $i_k$ for a given user $k$ depends implicitly on $s_k$ (for MF) as well as on all the other users codewords and powers $s_{\ell}, p_{\ell}, \forall \ell \neq k$, but does not depend on user $k$’s power.

Our goal in this setup is to derive a distributed algorithm in which individual users adjust their corresponding codewords and powers to meet specified target SINRs \( \{\gamma_1^*, \gamma_2^*, \ldots, \gamma_K^*\} \) that must be admissible as defined in [24] and satisfy
\[ \sum_{k=1}^K \frac{\gamma_k^*}{1 + \gamma_k^*} < N \]  

We note that the admissibility condition (9) is derived for ideal user channels in [24] but it extends in a straightforward way to the multipath channel scenario1.

III. JOINT CODEWORD AND POWER ADAPTATION AS A SEPARABLE GAME

A separable game is a particular type of non-cooperative game [9] in which the player cost functions are separable with respect to variables that define user strategies [20]. For the uplink CDMA scenario with multipath described in Section II players are the active users in the system and their corresponding strategies are adaptation of their codewords and powers with strategy spaces formally defined by the $N$-dimensional sphere with radius 1 for codewords
\[ S_k = \{ s_k | s_k \in \mathbb{R}^N, \|s_k\| = 1 \} \quad \forall k = 1, \ldots, K \]  

and by the set corresponding to the real interval $(0, P_{\text{sup}}]$ for powers
\[ P_k = \{ p_k | p_k \in (0, P_{\text{sup}}] \} \quad \forall k = 1, \ldots, K \]

where $P_{\text{sup}}$ is the maximum power level.

Following [7] we take the cost function of a given user $k$ to be the product between its power and its corresponding interference function
\[ u_k = p_k i_k = p_k s_k^T R_k s_k \quad \forall k = 1, \ldots, K. \]

which is separable with respect to the two parameters that define the user strategy – the corresponding codeword and power – and implies a separable game with the following components:
1) $K = \{1, \ldots, K\}$ is the set of players which are the active users in the system.
2) $S_k$ is the set of codeword strategies for player $k$ in (10).
3) $P_k$ is the set of power strategies for player $k$ in (11).
4) $u_k : S \times P \rightarrow (0, \infty)$ is the user cost function that maps the joint strategy spaces $S = S_1 \times \ldots \times S_K$ and $P = P_1 \times \ldots \times P_K$ to the set of positive real numbers.

The two sub-games of this game consist of codeword and power adaptation that minimize the user cost functions subject to the specified SINR constraints for which optimal strategies are [7]:

- Codeword update strategy
\[ \min_{s_k} u_k \big|_{P=\text{fixed}} \quad \text{subject to} \quad \gamma_k = \gamma_k^* \quad \forall k \]

- Power update strategy
\[ \min_{p_k} u_k \big|_{S=\text{fixed}} \quad \text{subject to} \quad \gamma_k = \gamma_k^* \quad \forall k \]

These two sub-games are convex and have at least one Nash equilibrium which implies existence of a Nash equilibrium for the joint codeword and power adaptation game [7], [20]. At

1The conservation law for the MMSE receiver used for proving (9) in [24] is applicable to the multipath channel scenario considered in our paper but we omit the proof due to space constraints.
the optimal Nash equilibrium codeword and power strategies satisfy:

- The optimal codeword update strategy is a greedy IA procedure [14] that consist of replacing the current codeword of a given user with the eigenvector corresponding to the minimum eigenvalue\(^2\) of the interference+noise correlation matrix \(R_k\) in equation (6) and satisfies also the determinant condition [7]

\[
D_k^* = (-1) \begin{vmatrix} 2p_k(R_k - \gamma_k^2 I_N) & 2s_k \\ 2s_k & 0 \end{vmatrix} > 0 \quad (15)
\]

- The optimal power strategy is to update power to match the specified target SINR for users.

These strategies imply that at a Nash equilibrium all codewords \(s_k^*\) are minimum eigenvectors of corresponding interference+noise correlation matrices \(R_k\), that is

\[
R_k s_k^* = \lambda_k s_k^* \quad \forall k = 1, \ldots, K \quad (16)
\]

with \(\lambda_k\) being the minimum eigenvalue of \(R_k\). The value of user \(k\) SINR at the optimal Nash equilibrium is equal to the target SINR and is expressed as

\[
\gamma_k^* = \frac{p_k}{\lambda_k} \quad \forall k = 1, \ldots, K \quad (17)
\]

and is achieved with minimum power since \(\lambda_k\) is the minimum value that the denominator in the SINR expression (7) can take.

IV. THE ALGORITHM FOR JOINT CODEWORD AND POWER UPDATES USING INCREMENTAL STRATEGIES

The strategies that define the optimal Nash equilibrium solution of the joint codeword and power adaptation game discussed in the previous section may lead to abrupt changes of the user codeword and/or power which are not desirable in practical implementations and similar to [7] the proposed algorithm uses incremental updates:

- Codeword update of user \(k\) at step \(n\) of the algorithm is:

\[
s_k(n + 1) = \frac{s_k(n) + m_\beta x_k(n)}{||s_k(n) + m_\beta x_k(n)||} \quad (18)
\]

where \(x_k\) is the minimum eigenvector of corresponding interference+noise correlation matrix \(R_k\), \(\beta\) is a parameter that limits the how far in terms of Euclidian distance the updated codeword can be from the old codeword, and \(m = \text{sgn} [s_k(n)x_k(n)]\).

- Power update of user \(k\) at step \(n\) of the algorithm is:

\[
p_k(n + 1) = p_k(n) - \mu(p_k(n) - \gamma_k^* i_k(n)) \quad (19)
\]

where \(0 < \mu < 1\).

The algorithm for joint codeword and power adaptation consists of two distinct stages carried out sequentially by active users in the system: one in which users perform incremental adaptation of their codeword followed by incremental adaptation of their power. The algorithm is formally stated below:

1) Input Data:

- Codewords \(s_k\), powers \(p_k\), channel matrices \(H_k\), and target SINRs \(\gamma_k^*\) for active users \(k = 1, \ldots, K\).
- Noise covariance matrix \(W\)
- Constants \(\beta, \mu\), and tolerance \(\epsilon\).

2) IF admissibility condition in equation (9) is satisfied GO TO Step 3. OTHERWISE STOP, the desired system configuration is not admissible.

3) FOR each user \(k = 1, \ldots, K\) DO

a) Compute corresponding \(R_k(n)\) using equation (6) and determine its minimum eigenvector \(x_k(n)\).

b) Update user \(k\)’s codeword using equation (18).

c) Update user \(k\)’s power using equation (19).

4) IF change in cost function is larger than \(\epsilon\) for any user then GO TO Step 3 OTHERWISE a Nash equilibrium is reached.

5) IF optimality condition (15) is true then STOP: an optimal Nash equilibrium has been reached. OTHERWISE GO TO Step 3.

The check of the optimality condition (15) in Step 5 ensures that the optimal Nash equilibrium is reached and that the algorithm does not stop in a sub-optimal fixed point. We note that, numerically, a fixed point of the algorithm may be reached when the codeword and power updates result in decreases of the user cost functions that are smaller than the specified tolerance, but if the optimality condition (15) is not satisfied the return to Step 3 and the incremental updates that will follow will move the system away from the sub-optimal Nash equilibrium toward to optimal one. We also note that, as it is the case with incremental/adaptive algorithms, convergence speed of the algorithm depends on the values of the corresponding increments specified by the algorithm constants \(\mu\) and \(\beta\).

The proposed algorithm may be run in a centralized manner at the base station receiver, or in a distributed way where individual users update codeword and power using feedback from the receiver [6], [18]. In order for user \(k\) to perform the updates the interference+noise correlation matrix \(R_k\) is needed, which can be obtained from the correlation matrix of received signal \(R\) by subtracting its contribution \((p_k H_k s_k s_k^T H_k^T)\) as shown by equation (6). Thus, if the correlation matrix of the received signal \(R\) is made available to individual users through a feedback channel [6], [18] the proposed algorithm can work in distributed manner. We note that the amount of feedback information needed for distributed CDMA codeword adaptation is still a mostly open problem and only few empirical studies have been performed to date [11], [13]. For the proposed algorithm, feedback was not explicitly considered and we assumed that all users have instantaneous access to matrix \(R\). A thorough investigation of the proposed algorithm with feedback in the context of practical scenarios will be the object of future research.

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\(^2\)This is also referred to as the minimum eigenvector.
V. SIMULATIONS AND NUMERICAL EXAMPLES

We considered a CDMA system with $K = 5$ users in a signal space of dimension $N = 4$ and white noise with covariance matrix $W = 0.1I_4$. The algorithm constants are $\beta = 0.02$, $\mu = 0.01$, and tolerance $\epsilon = 0.01$ and diagonal matrices were chosen randomly for all users:

\[
\begin{align*}
H_1 &= \text{diag}(0.7948, 0.9797, 0.1365, 0.6614) \\
H_2 &= \text{diag}(0.2844, 0.4235, 0.5798, 0.3798) \\
H_3 &= \text{diag}(0.7833, 0.0592, 0.8744, 0.7889) \\
H_4 &= \text{diag}(0.4387, 0.9601, 0.4399, 0.6288) \\
H_5 &= \text{diag}(0.1338, 0.5751, 0.0129, 0.6124)
\end{align*}
\]

Initial user powers are $P_\ell = 0.1$, $\forall \ell$ and initial codewords were selected randomly.

In the first example we illustrate convergence of the algorithm for user target SINRs $\gamma^* = \{5.0, 4.0, 3.0, 2.0, 1.0\}$ which are admissible in the system. Figure 1 shows the variation of the user SINRs and powers until the algorithm converges. We note that the initial sharp increase in SINRs and powers followed by smooth adaptation and convergence to the specified target SINR values which are achieved with minimum user powers. We also note that for this example the gradient-based algorithm [12] does not converge since it is trapped in a sub-optimal point. The user codewords and powers yielded by the proposed algorithm for this example are

\[
\begin{align*}
s_1 &= 0.9678, s_2 = -0.0581, s_3 = -0.3448, s_4 = 0.0302, s_5 = 0.0476 \\
s_6 &= -0.1246, s_7 = -0.0548, s_8 = -0.0175, s_9 = 0.9840, s_{10} = 0.6841 \\
s_11 &= -0.0110, s_12 = 0.9767, s_13 = -0.3777, s_14 = -0.0385, s_15 = 0.0053 \\
s_16 &= 0.2186, s_17 = 0.1993, s_18 = 0.8592, s_19 = -0.1711, s_20 = 0.7278
\end{align*}
\]

which imply that the desired target SINRs are satisfied with a tolerance of $O(\epsilon)$. The correlation matrix of the received signal for this example, computed using equation (5), is

\[
R = \text{diag}(0.8381, 0.9913, 0.8579, 1.2758)
\]

We note that all users appear to put most of their transmitted signal energy over those signal dimensions with larger channel gains avoiding essentially those dimensions with small channel gains. This property has been observed in all simulations that we performed and will be the object of future investigations aimed at establishing properties of optimal Nash equilibria in terms of user channel matrices and target SINR values.

In the second example we illustrate the tracking ability of the proposed incremental algorithm and assume that once the considered system has reached the optimal Nash equilibrium described above user 5 changes its SINR to two new values $\gamma^*_5 = 2.5$ and $1.75$ at two distinct time instances. Figure 2 displays the variation of the user SINRs and powers for this example showing how the algorithm tracks the variable SINR of user 5. Thus, the proposed algorithm can be used in dynamic systems with changing QoS requirements for users.

VI. CONCLUSION

In this paper we presented a new algorithm for joint codeword and power adpatation in uplink CDMA systems with multipath channels between users and base station. The proposed algorithm uses incremental codeword and power updates in the direction of the optimal strategies that minimize cost functions and has a smooth transition to a steady state configuration where specified SINR targets are achieved with minimum power. The algorithm can also track variable target QoS requirements.
SINRs for active users in the system and is useful for dynamic wireless system with changing QoS requirements.

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