Interference Avoidance With Limited Feedback

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Abstract—Interference avoidance (IA) is an adaptive modulation technique by which transmitters in a wireless communication system are optimized using feedback from the receiver in order to minimize interference and better suit the environment in which they operate. In this paper we study IA algorithms in the context of limited feedback and evaluate system performance as a function with which the interference information is quantized for transmission over the feedback channel. Numerical results obtained from extensive simulations indicate that non-uniform scalar quantization requiring as little as 2 bits per scalar value for encoding imply performance very close to that when no quantization is applied.

I. INTRODUCTION

IA has emerged in the literature as a class of methods that enable distributed transmitter optimization in wireless systems to better suit the environment in which the systems operate under specified Quality of Service (QoS) requirements [1]. IA-based transmitter adaptation uses information about the interference corrupting the desired signal at the receiver that is acquired over a feedback channel, and the transmitter is updated in response to changing patterns of interference. We note that so far IA algorithms have been investigated under the assumption of unlimited feedback from the receiver, in which case the transmitter has instantaneous access to the real interference information at the receiver. However, in practice only limited feedback is available, and the amount of interference information that the receiver may feed back to the transmitter is limited by the capacity of the feedback channel which determines the precision with which the interference information needs to be quantized.

In this paper we study IA algorithms in the context of quantized interference information and our objective is to characterize system performance as a function of the quantization accuracy. Specifically, we consider the uplink of a Code Division Multiple Access (CDMA) system and look at the sum capacity of the corresponding vector multiple access channel model as a global measure of system performance when quantized interference information is used in the codeword adaptation process.

The paper is organized as follows: in Section II we present the system model and the greedy IA algorithm, followed by a discussion on quantizing interference information with finite precision in Section III. In Section IV we describe the quantization experiments performed and present numerical examples obtained from simulations. Final conclusions are presented in Section V.

II. SYSTEM MODEL AND THE IA ALGORITHM

We consider the uplink of a synchronous CDMA system with $K$ active users in a signal space of dimension $N$ implied by finite common bandwidth and signaling interval constraints [2], for which the $N$-dimensional received signal vector at the base station corresponding to one signaling interval is given by the expression

$$ r = \sum_{k=1}^{K} b_k \sqrt{p_k} s_k + n $$

where $s_k$ is the $N$-dimensional unit-norm codeword corresponding to user $k$, $b_k$ is the information symbol transmitted by user $k$, $p_k$ is the received power at the base station for user $k$, and $n$ is the additive Gaussian noise that corrupts the received signal with zero-mean and covariance matrix $W = E[nn^\top] = \sigma^2 I_N$. By defining the $N \times K$ codeword matrix, $S = [s_1, \ldots, s_k, \ldots, s_K]$ having as columns the user codewords and the $K \times K$ diagonal matrix of received user powers $P = \text{diag}[p_1, \ldots, p_k, \ldots, p_K]$ we can rewrite the received signal in a compact matrix-vector form as:

$$ r = SP^{1/2}b + n $$

where $b = [b_1 \ldots b_K]^\top$ is the vector containing the information symbols sent by users.

At the receiver matched filters are used to decode symbols transmitted by users, and the decision variable for user $k$ is given by

$$ d_k = s_k^\top r = b_k \sqrt{p_k} + s_k^\top \left( \sum_{\ell=1, \ell \neq k}^{K} b_\ell \sqrt{p_\ell} s_\ell + n \right) $$

for which the signal-to-interference plus noise-ratio (SINR) corresponding to user $k$ is expressed as

$$ \gamma_k = \frac{p_k}{s_k^\top R_k s_k} $$

where

$$ R_k = \sum_{\ell=1, \ell \neq k}^{K} p_\ell s_\ell s_\ell^\top + W $$

is the correlation matrix of the interference+noise seen by user $k$. We note that $R_k$ can be rewritten in terms of the correlation matrix $R$ of the received signal in equation (2) as

$$ R_k = \frac{SPS^\top + W}{R} - p_k s_k s_k^\top = R - p_k s_k s_k^\top $$
In this setup, greedy IA [1] consists of replacing the current codeword of a given user \( k \) with the minimum eigenvector \( x_k \) of \( R_k \) and minimizes the effective interference corrupting user \( k \)’s signal at the receiver. The IA algorithm is formally stated here:

**The Greedy IA Algorithm**

1) Initialize user codeword and power matrices \( S \) and \( P \), and set the noise covariance matrix \( W \)
2) For \( k = 1, \ldots, K \)
   a) Compute matrix \( R_k \) using equation (6)
   b) Replace current codeword of user \( k \) with the minimum eigenvector of \( R_k \)
3) Repeat step 2 until a fixed point is reached.

Numerically, a fixed point of the algorithm is defined with respect to a stopping criterion, and is reached when the difference between two consecutive values of the stopping criterion is within a specified tolerance \( \epsilon \). The stopping criterion can be related to individual users like the codeword SINR or the Euclidian distance between codewords and their corresponding replacements, or to global system measures like sum capacity or total squared correlation. Convergence of the greedy IA algorithm has been investigated analytically in [1], [3], where it has been shown that it converges to generalized Welch Bound Equality (WBE) sequences which maximize the sum capacity in the corresponding vector multiple access channel model [4].

A distributed implementation of the greedy IA algorithm in which users in the system update their corresponding codewords individually does not require complete knowledge by a given user about all the other active users in the system (in terms of codewords and powers). Rather, the only piece of information needed by an individual user \( k \) to apply the greedy IA algorithm is the correlation matrix \( R_k \) of the interference+noise that affects its transmitted signal at the receiver, which can be obtained by subtracting user \( k \)’s contribution from the received signal correlation matrix \( R \) as shown by equation (6). Thus, distributed implementation of the greedy IA algorithm requires that only the correlation matrix of the received signal \( R \) be made available through the feedback channel.

**III. QUANTIZING INTERFERENCE INFORMATION**

As mentioned in the previous section, the interference information that users need to perform the greedy IA codeword adaptation is embedded in the received signal correlation matrix \( R \). We note that \( R \) is an \( N \times N \) symmetric matrix, and that the values of only \( N(N+1)/2 \) elements need to be actually transmitted over the feedback channel.

In our study of the greedy IA algorithm we consider scalar quantization [5, Sec. 6.5.1] of the \( R \) matrix and assume that the quantized matrix \( \hat{R} \) (whose elements are the quantized version of the corresponding elements in \( R \)) is used during the IA algorithm to compute the interference+noise correlation matrices of active users in the system, that is

\[
\hat{R}_k = \hat{R} - p_k s_k s_k^\top
\]  

(7)

is used instead of \( R_k \).

In scalar quantization the set of real numbers \( \mathbb{R} \) is partitioned into \( L \) disjoint subsets \( \{R_k\}_{k=1}^L \) and a representation point is chosen for each subset. The quantization function is

\[
Q(x) = \hat{x}_i \quad \forall x \in \mathcal{R}_i
\]  

(8)

and is nonlinear and non-invertible. For \( L \) quantization levels a number \( B = \log_2 L \) of bits are sufficient to encode them into a binary sequence, and usually \( L \) is chosen to be a power of 2. Depending on how the width of the quantization regions \( \mathcal{R}_i \) is chosen one may have uniform and non-uniform quantizers. Uniform quantizers are the simplest type of quantizers in which quantization regions \( \{\mathcal{R}_i\}_{i=1}^{L-1} \) are equal to some \( \Delta \), called quantization width. The design of a uniform quantizer for random variables is done mainly by numerical techniques to minimize the squared error distortion \( D = E[(X - Q(X))^2] \).

In the case of non-uniform quantizers, the condition that quantized regions be equal is not imposed which implies fewer constraints in the minimization of the distortion. The quantizer is designed based on the necessary conditions for optimality, known as Lloyd-Max conditions [5].

We formally state the IA algorithm with quantized interference information here:

**The Greedy IA Algorithm with Quantized Information**

1) Input data: noise covariance matrix \( W \), initial user codeword and power matrices \( S \) and \( P \), tolerance \( \epsilon \).
2) For \( k = 1, \ldots, K \)
   a) Compute true \( R \) matrix at the receiver and quantize it to obtain matrix \( \hat{R} \).
   b) Compute matrix \( \hat{R}_k \) using equation (7).
   c) Replace current codeword of user \( k \) with the minimum eigenvector of \( \hat{R}_k \).
3) Repeat step 2 until a fixed point is reached.

We note that the analytical approach in [1], [3] does not ensure convergence of the greedy IA algorithm with quantized information to a fixed point, and we will use the sum capacity of the corresponding vector multiple access channel to establish empirically the convergence. When the quantized received signal correlation matrix \( \hat{R} \) is used this has the expression [4]

\[
C_{\text{sum}} = \frac{1}{2} \log |\hat{R}| - \frac{1}{2} \log |W|
\]  

(9)

and we say that a fixed point of the algorithm is reached when the difference between two consecutive values of the sum capacity is within the specified tolerance \( \epsilon \).

**IV. SIMULATIONS AND NUMERICAL RESULTS**

In order to evaluate the performance IA algorithms with quantized interference information we consider a system with
$K = 15$ users in a signal space of dimension $N = 10$ and additive white Gaussian noise with zero mean and covariance matrix $W = 0.1I_{10}$. We assume that all users have the same power $p_k = 1, \forall k$, and perform experiments in which uniform and nonuniform quantization are used. These are applied to quantization of the received signal correlation matrix, and the performance of the system is evaluated using the sum capacity which is computed as described in Section III. For each experiment we perform 100 runs of the greedy IA algorithm with quantized information starting with random initial user codewords, and the resulting variation in sum capacity is compared with the sum capacity variation corresponding to the greedy IA algorithm described in Section II and plotted for reference in Figure 1. We note that in this case all runs converge to the maximum sum capacity value.

In the case of uniform quantization we assume that the entries of the $R$ matrix have a normal distribution, and we used the quantizer parameters in [5, Table 6.2] for 8, 16, and 32 quantization levels that are encoded on 3, 4, and respectively 5 bits. Results of this experiment are presented in Figure 2. We note that in this case the algorithm does not converge when 8 quantization levels are used. We also note that, although the algorithm converges in terms of sum capacity for 16 and 32 quantization levels, distinct runs result in different values of the sum capacity and the average of these final values is less than the maximum sum capacity, which is unlike the algorithm that employs the true (unquantized) information.

In the case of non-uniform quantization we used the values of the true (unquantized) correlation matrix $R$ that occurred during the 100 runs of the greedy IA algorithm described in Section II to develop non-uniform quantizers with 4, 8, 16, and 32 levels using the Lloyd-Max conditions. Results of this experiment are presented in Figure 3. We note that in this case the algorithm converges even when only 4 quantization levels are used, and that the sum capacity values achieved in this case are much closer to the maximum sum capacity achievable with unquantized information.
(a) 5-bit nonuniform quantization
(b) 4-bit nonuniform quantization
(c) 3-bit nonuniform quantization
(d) 2-bit nonuniform quantization

Fig. 3. Variation of sum capacity for the greedy IA algorithm with nonuniform quantization of the interference information.

V. CONCLUSION

In this paper we performed an empirical study of IA algorithms with limited feedback when only quantized interference information is available for codeword adaptation. We evaluated convergence of the greedy IA algorithm with quantized information and looked at system performance in terms of the sum capacity of the corresponding vector multiple access channel. Our results have shown that non-uniform quantization implies better performance than uniform quantization, and that using as little as 4 nonuniform quantization levels results in good system performance.

REFERENCES