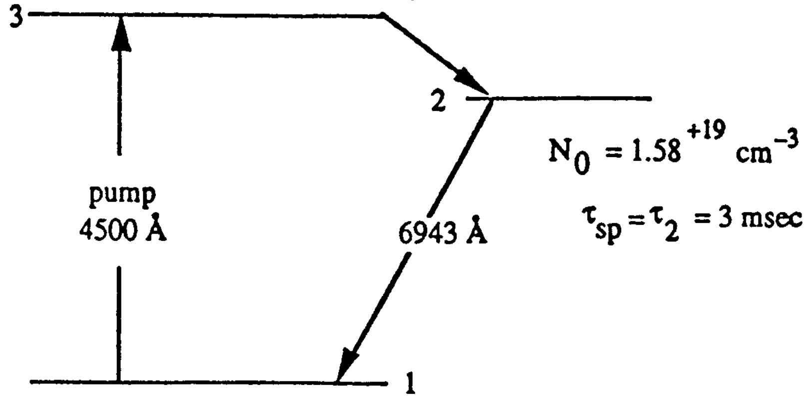
9.17

One should always start each problem with an energy level diagram showing how the atoms get to the upper state and how the atoms return. **The most important fact is that atoms must be conserved and thus:**

$N_1 + N_2 = N_0 = 1.58\times10^{19}\ \text{cm}^{-3}$ always



For $N_2 = 10^{19}\ \text{cm}^{-3}$, $N_1 = 0.58\times10^{+19}\ \text{cm}^{-3}$ assuming that nothing resides in state 3.

(b) $\quad P_{spont} = \dfrac{N_2 V}{\tau_{sp}} \cdot h\nu_{laser} = 14.3\ \text{kW};$

This power must be supplied by absorption on the $1 \to 3$ route. In the best of all worlds, all of the atoms pumped to 3 would transfer to state 2 – the fraction that does so is called the quantum efficiency of the pump. For ruby, that efficiency is 70%, but for this problem, we can assume it to be 100%

(a) $\quad P_{pump} = \dfrac{\lambda_{31}}{\lambda_{21}} P_{spont} = 22.1\ \text{kW}.$ (It would be 31.6 kW for the real system)

$$\tau_{R.T.} = \frac{2 l_g n_g}{c} + \frac{2(d - l_g)}{c} = \frac{2\text{x}15\text{x}1.78}{c} + \frac{2\text{x}5}{c} = 2.1\times10^{-9}\ \text{s};$$

$$2g_{th} = \ln\frac{1}{R_1} + \ln\frac{1}{R_2} = 5.13\times10^{-2} + 0.357 = 0.408$$

The first log term is internal loss (per round-trip), last is the external value. The photon lifetime is given by:

$$\frac{1}{\tau_p} = \frac{<\alpha_T>2d}{\tau_{R.T.}} = 19.4^{+9}\ \text{s}^{-1};\ \tau_p = 5.15\ \text{ns};\ \eta_{cpl} = 0.357/0.408 = 0.875$$

$$P(\max) = \eta_{cpl} \cdot \frac{h\nu}{\tau_p}\left\{\frac{n_i - n_{th}}{2} - \frac{n_{th}}{2}\ln\left(\frac{n_i}{n_{th}}\right)\right\}$$

$n_i = 15\ \text{cm} \times 1\ \text{cm}^2 \times [(10^{19} - 0.58\text{x}10^{19})\text{cm}^{-3}] = 6.3\times10^{+19}$ atoms

$\gamma_{th} = \dfrac{1}{2l_g}\ln\dfrac{1}{R_1 R_2} = 1.36^{-2}\ \text{cm}^{-1} = (N_2 - N_1)_{th}\,\sigma;$

$(N_2 - N_1)_{th} \cdot A \cdot l_g = n_{th} = 1.61^{+19}$ atoms; Thus $n_1/n_{th} = 3.92$; and $\eta_{xtn} = 0.98$;

(c) Hence $P(\max) = \dfrac{0.351}{0.408} \cdot \dfrac{2.86\times10^{-19}}{5.15\times10^{-9}} \left\{1.24\times10^{+19}\right\} = 603\ \text{MW}$

(d) $W_{out} = \eta_{cpl}\eta_{xtn} \cdot (h\nu n_i/2) = 7.72$ Joules