ECE 476/576

HW #5

7.6

Assume T = 300K and M = 22 AMU, 1.0 AMU = 1.67^{-27} kg;
$\Delta\nu_D = (8kT \ln 2/Mc^2)^{1/2}\nu_0 = 3.17 \times 10^{-6}\,\nu_0$

λ	ν_0	$\Delta\nu_D$	$\Delta\lambda_D$
6328Å	4.74^{14} Hz	1.5GHz	0.02Å
1.1523μm	2.6^{14}	0.823	0.0365Å
3.39μm	8.85^{13}	0.28	0.107Å

7.10

This is a reasonable model for the pulsed N_2 laser which has a very high gain and lases at 0.337 μm. Its major drawback is that it is self-terminating because the inversion can not be maintained indefinitely – fact that comes out of the analysis that follows.

$\frac{dN_2}{dt} = P_2 - \frac{N_2}{\tau_2}$ whose solution is: $N_2(t) = P_2\tau_2(1 - \exp[-t/\tau_2])$

$\frac{dN_1}{dt} = +\frac{N_2}{\tau_2} - \frac{N_1}{\tau_1}$ or $\frac{dN_1}{dt} + \frac{N_1}{\tau_1} = P_2(1 - \exp[-t/\tau_2])$

$$N_1 = P_2\tau_1\left\{1 - \frac{\tau_1/\tau_2}{\tau_1/\tau_2 - 1}e^{-t/\tau_1} + \frac{1}{\tau_1/\tau_2 - 1}e^{-t/\tau_2}\right\}$$

It is informative to consider the steady-state populations, i.e., where $t >> \tau_{2,1}$. $N_2(t\to\infty) = P_2\tau_2$ and $N_1(t\to\infty) = P_2\tau_1$. Note that since $\tau_1 > \tau_2$, we have the undesirable situation of $N_1 > N_2$ which means that the system will not lase in a steady state. However, one can obtain a transient inversion (and a laser) as the sketch below indicates.

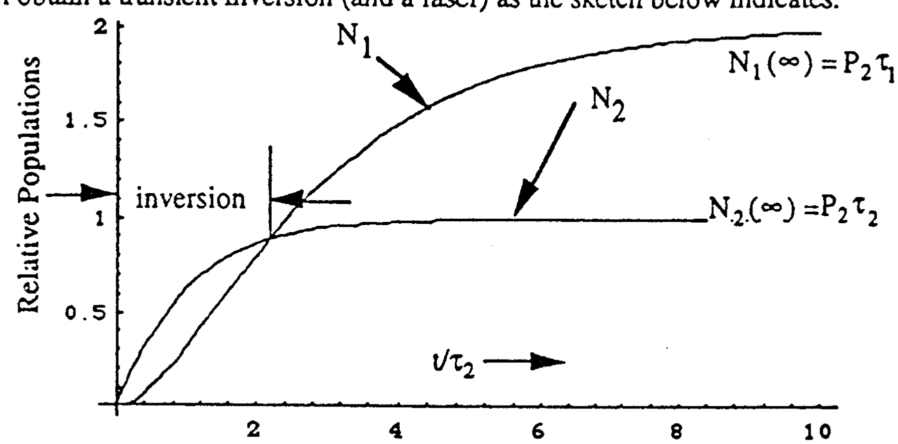

Figure for problem 7.10

$N_2(t\to\infty) = 10^{20} \times 10^{-6} = 10^{14}\ \text{cm}^{-3}$, $N_1(t\to\infty) = 10^{20} \times 2 \times 10^{-6} = 2 \times 10^{14}\ \text{cm}^{-3}$
Let $t/\tau_1 = x$; $(\tau_1/\tau_2) = 2$; $N_1 = N_2$ when $2(1 - 2e^{-x} + e^{-2x}) = (1 - e^{-2x})$;

Collect terms, multiply by e^{2x} and factor: $f(x) = [e^x - 3][e^x - 1] = 0$;
where $x = t/\tau_1$; F(x) = 0 at x = 0 (i.e. at the start) or $x = t/\tau_1 = \ln 3$: ∴ $t = 1.0986\tau_1 = 2.2$ μsec